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Greenbelt, Maryland 20771

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STUDY OF LIQUID SLOSH IN THE TRACKING AND DATA RELAY SATELLITE HYDRAZINE TANKS

by


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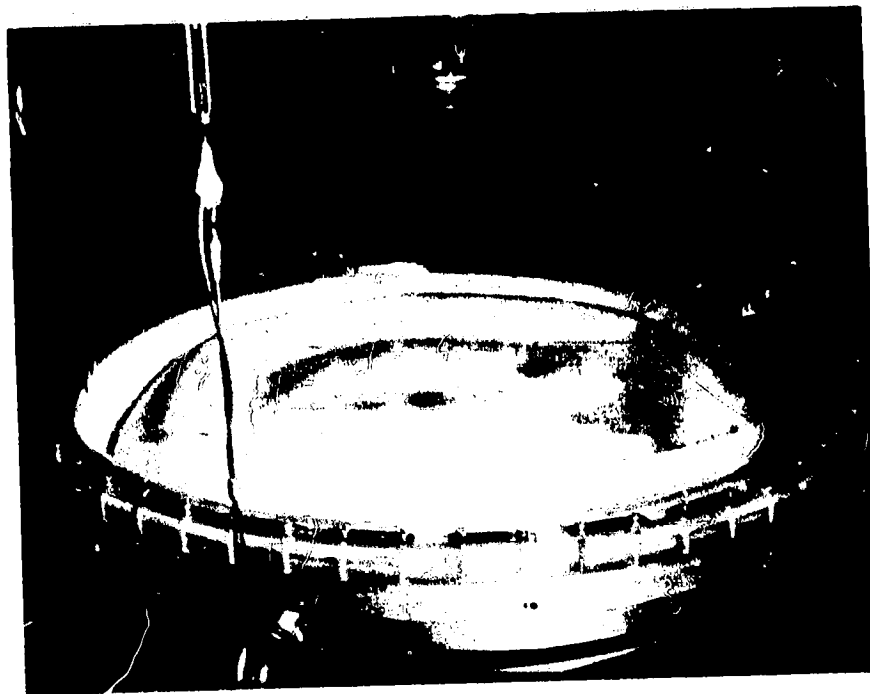
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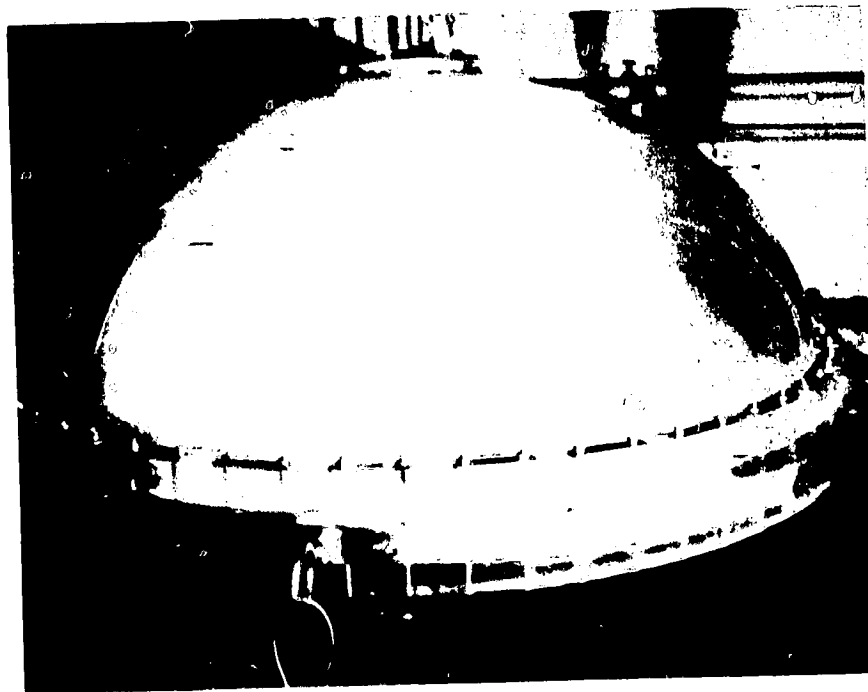
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ORIGINAL PAGE
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a. Liquid Under Ullage



b. Liquid Over Ullage

FRONTISPIECE
TDRSS PROPELLANT TANK CONFIGURATIONS

ABSTRACT

An experimental study was conducted to provide data for evaluation of the parameters for an analytical mechanical model representation of liquid/interface dynamics in the TDRSS propellant tanks. Models were developed for two liquid-under-ullage (forward tank) configurations and for one liquid-over-ullage (aft tank) configuration. However, additional test runs were conducted with liquids of different densities in both cases to allow separation of bladder stiffness and gravity effects under various simulated steady acceleration conditions. Both static and dynamic parameters are evaluated to provide a good prediction of observed results.

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I. INTRODUCTION

This report presents the results of a study of the slosh dynamics and development of an analytical model for the hydrazine tanks of the Tracking and Data Relay Satellite. The two ellipsoidal tanks of this satellite are quite unique, in that they employ a polymeric bladder to separate the propellant from the ullage. Furthermore, as mounted in the satellite relative to the thrust vector, one tank is oriented with liquid under ullage, and the other with liquid over ullage, as shown in the two photographs of the Frontispiece. The design allows for positive expulsion of the propellant by gas pressurization of the ullage for any orientation. Although this is a simple and reliable design functionally, the dynamic properties of the bladder/liquid interface in this particular geometry were unknown. In spite of the wealth of knowledge [1] that is available from previous work on propellant slosh in tanks of various designs, there was only one published report [2] on experience with a tank having a bladder. Furthermore, the geometry and orientation of the TDRSS tanks was significantly different from that tank. Hence the program described herein was necessary for proper design of the TDRSS system. The resulting data should be considered as applicable to the specific TDRSS configuration only, and under no circumstances used for extrapolation to any other size or configuration tank.

At the outset of this program, considerable analytical thought already had been given to the potential slosh response. In fact, a tentative analytical model had even been formulated. However, it eventually became apparent that the problem involved uncertainties to such an extent that the conduct of a preliminary series of experiments became mandatory in order to provide more physical insight into the model formulation. The preliminary experiments were designed to investigate both static and dynamic behavior of several propellant fill depth and geometrical orientations of the tank. The particular fill depth and orientation configurations were chosen to coincide with those anticipated to occur at particular events on the flight trajectory. A special, full-size plexiglass tank with bladder installed was provided for the study. Sufficient photographic observations and other dynamic response data were acquired to allow a determination of the adequacy of the existing postulated analytical model. As the program progressed, it became apparent that significant modification to the original analytical model was appropriate. Formulation of the numerical values for the various parameters of this model is the final objective of this program, as described herein. Therefore, we begin with a description of the final model concept, and thereafter describe how the experimental results provided the required information for a complete specification of a model for each of the TDRSS tanks.

II. ANALYTICAL MODEL CONCEPTS

Preliminary tests showed that sloshing in the forward tank (liquid -under-ullage configuration) was similar to that in a nonbladdered tank of the same shape. Further, the liquid and bladder were symmetrically arranged around the vertical centerline. Sloshing in the aft tank (liquid -over-ullage) was, however, considerably different from the forward tank, primarily because most of the liquid was held on one side of the vertical centerline by the bladder; see the frontispiece. Thus, two analytical models are required.

A. Liquid-Under-Ullage Model (Forward Tank)

The slosh model shown in Figure 1a for the forward tank is based on the model for a corresponding nonbladdered tank (1). A pendulous mass m_1 is used to represent the liquid participating in the sloshing motion, and an immobile mass m_0 is used to represent the remainder of the liquid. The pendulum pivot height h_1 is chosen so that the sloshing torques created by a rotational oscillation about the X-axis are duplicated, and the immobile mass is assigned a centroidal moment-of-inertia I_0 to duplicate the rigid-body-like rotation of the liquid. The immobile mass is positioned along the vertical centerline such that the center-of-mass location is the same for the liquid and the model. (Because of symmetry, a similar model is valid for the X-Z plane. In fact, the plane of the model is determined by the rotational and translational axes of the excitation.)

The primary conceptual difference between this model and a normal slosh model is the use of a torsional spring K_θ at the pendulum pivot which is needed to simulate the bladder restraint on the liquid motion. The slosh natural frequency is therefore

$$\omega_1 = \left(\frac{K_\theta}{M_1 L_1^2} + \frac{a}{L_1} \right)^{1/2} \quad (\text{II.1})$$

where a is the effective gravity or reversed steady Z-acceleration acting on the tank.

All the parameters of the model can be computed from the following kinds of test data.

(1) Center of mass location. (Note: the center of mass can instead be computed from the tank filling level.)

(2) Liquid force acting on the tank as a function of excitation frequency (i.e., resonance search data.) From these data, M_1 , ω_1 , C_θ can be computed.

(3) Liquid torque acting on the tank as a function of excitation frequency. I_0 and h_1 can be computed from these data; then, knowing the center of mass location, h_0 can be computed.

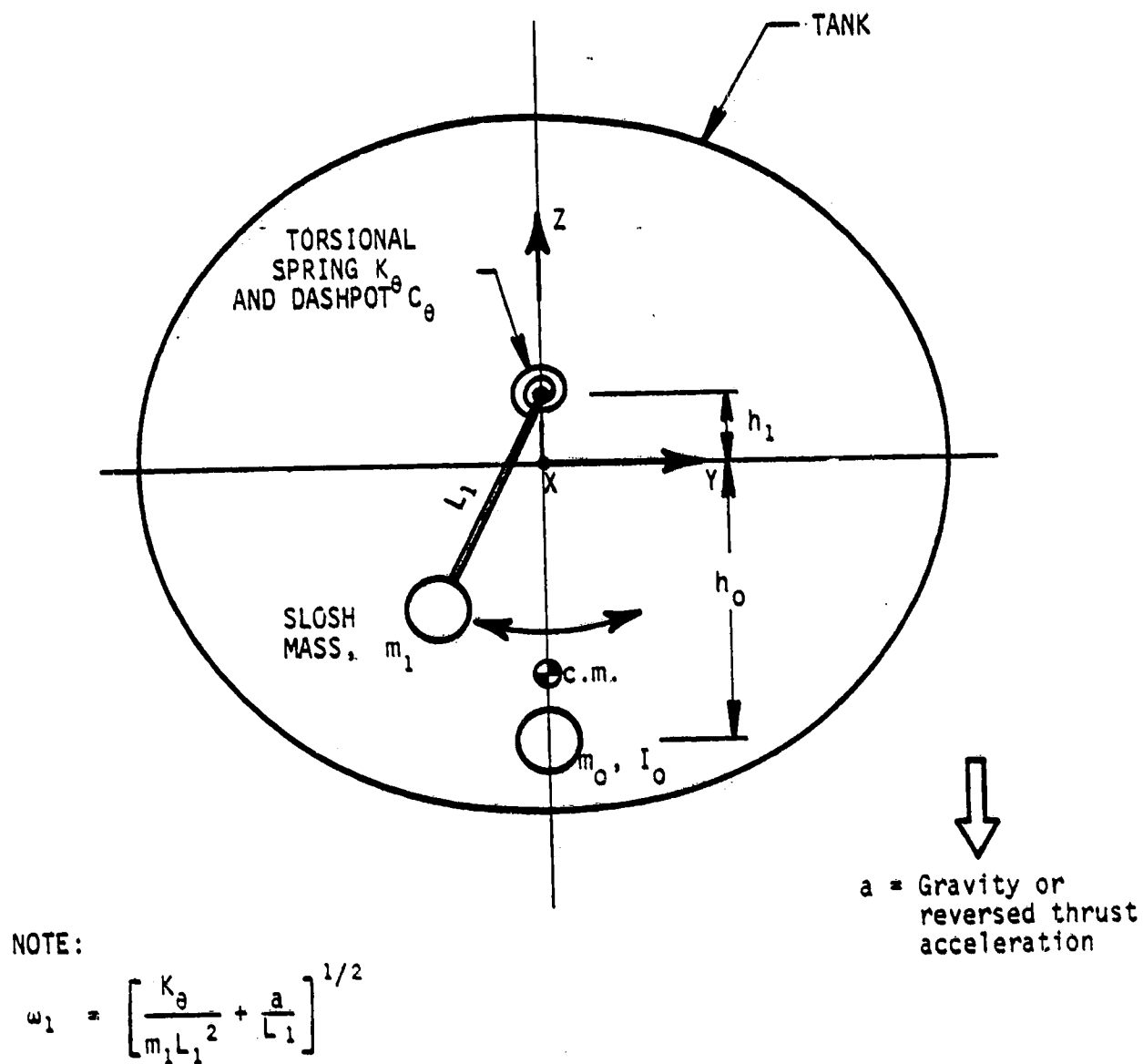


FIGURE 1a. MECHANICAL MODEL FOR LIQUID-UNDER-ULLAGE (FORWARD) TANK

(4) Values of ω_1 for several values of a . Thus, L_1 and K_θ can then be computed.

Since it is not practical in the laboratory to use centrifuges or drop-towers to obtain values of steady acceleration, a , that are different from normal gravity, the requirements of (4) cannot be met exactly. But as shown by the following discussion, a close approximation can be obtained, nonetheless, by using liquids of different densities in the tests. First, preliminary data showed that the shape of the bladder in the forward tank depends primarily on the volume of liquid under the bladder. This observation is consistent with the results of other tests of the bladder which showed that it was virtually inextensible and that its apparent stiffness (K_θ) was due to the bending stresses in the folds. Second, neither the bladder nor the liquid configuration will change when the liquid density changes, provided that the liquid volume stays the same. Therefore, it follows that the sloshing configuration (i.e., the mode shape) also should not change. The slosh mass m_1 would merely increase or decrease in proportion to the liquid density, and K_θ would not change since the fold pattern does not change. As can be seen from Equation II.1 data giving ω_1 for different values of m_1 can therefore be used to compute K_θ and L_1 individually. It should also be noted that the tests showed the damping of the liquid motions was due almost entirely to bladder viscoelasticity, with little or nothing contributed by the viscosity of the liquid. Thus, the use of various liquids does not result in any significant change in C_θ .

B. Liquid-Over-Ullage Model (Aft Tank)

Since sloshing in the forward tank is similar to conventional sloshing, most of the slosh model development and verification efforts were spent on the aft tank.

It was not apparent in advance what the slosh model for the aft tank should look like. Several possibilities could be made to fit the preliminary test data qualitatively. As mentioned in the Introduction, one form of analytical model had already been developed conceptually before the present work began. Since that model, with some modification, also fitted the preliminary data, it was chosen for further development. Figure 1b shows the configuration of the model.

Since the liquid is positioned mostly on one side of the vertical centerline, the slosh mass and immobile mass are also located on that side. The rest of the model can be understood most easily by summarizing some of the test observations.

First, a well defined slosh mode was excited by translational oscillations of the tank along the X-axis or by rotational oscillations about

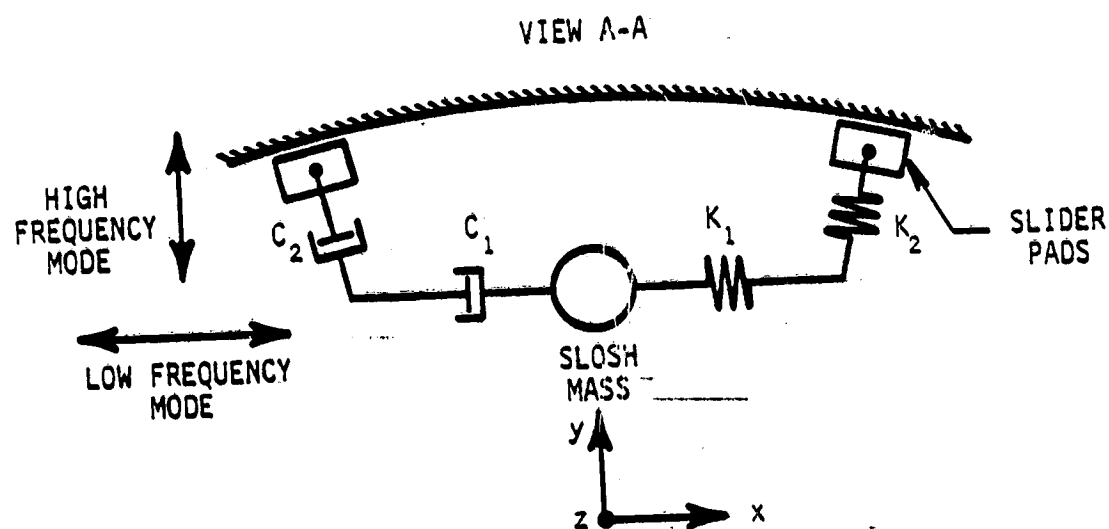
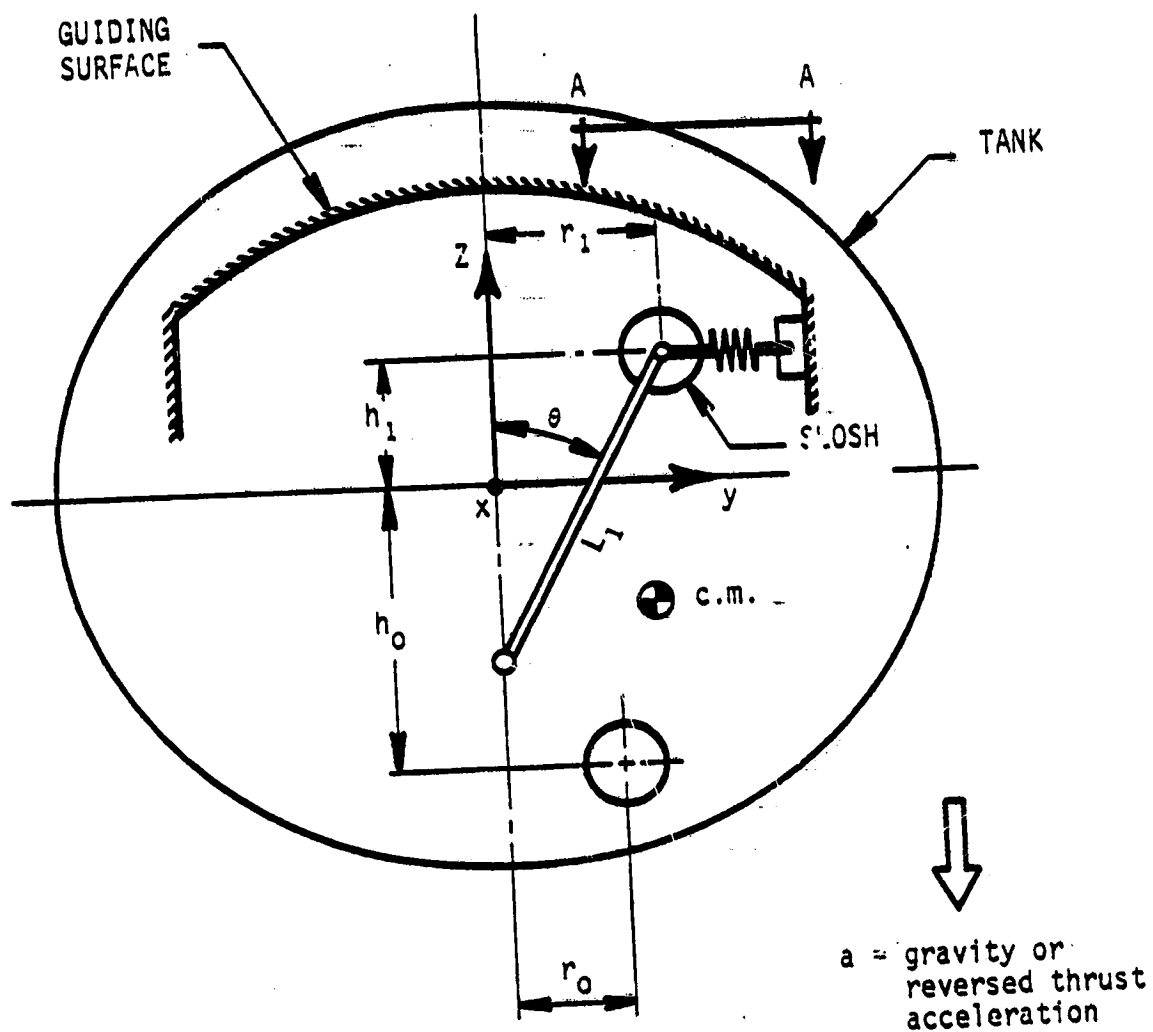


FIGURE 1b. MECHANICAL MODEL FOR LIQUID-OVER-ULLAGE (AFT) TANK

the Y or Z axis. A much higher frequency, but less well defined mode was excited by translations along the Y axis or by rotations about the X axis. (Because of the lack of symmetry, the X-Z plane is not equivalent to the Y-Z plane for the model.) Second, when the tank was rotated about the X-axis to simulate, for example, a change in direction of the thrust vector, the liquid would suddenly shift to the opposite half of the tank after some critical rotation angle was exceeded. When the tank was rotated about the Y axis, the liquid would shift slowly to some new position around the periphery of the tank after some critical angle was exceeded. To represent all these kinds of motions, the slosh mass m_1 is made part of an inverted pendulum that contacts an imaginary guiding surface through a system of springs, dashpots, and slider pads. For small oscillations of the tank, the "stiction" force that exists between the pads and the guiding surface is made large enough so that the pads remain stationary. The slosh mass is then free to oscillate to represent the appropriate slosh mode. As shown in View A-A, a spring K_1 , dashpot C_1 , and mass m_1 are selected such that the forces and torques due to the low frequency mode excited by tank translations along the X-axis and rotations about the Y and Z axes are duplicated.

Spring K_2 and dashpot C_2 are chosen similarly to represent the high frequency mode excited by translations along the Y axis and rotations about the X axis. (For this latter mode, the slosh mass is also given a centroidal moment of inertia I_{1x} , so that forces and moments as well as the natural frequency can be made to fit the test data.) In general, K_1 and K_2 depend on both the bladder stiffness and the steady acceleration a .

When the rotation about the Y-axis or the translation acceleration along the X-axis is large enough, the stiction force on the pads will be exceeded and the whole pendulum system will slide to a new position. The immobile mass is supposed to move in concert with the slosh mass for this type of motion. Likewise, when the rotation about the X-axis or the translational acceleration along the Y-axis is large enough, the inverted pendulum will suddenly swing through the vertical and come to rest at a symmetrical position on the other side of the centerline. (Again, the immobile mass moves with the slosh mass.) Further, the spring-dashpot-slider pads also rotate as the pendulum swings, always remaining in contact with the guiding surface; this provides a cushioning of the impact of the slosh mass when the pendulum comes to rest in its new position.

The test data required to validate the slosh model are similar to that listed previously for the forward tank. However, slosh force data are needed, in this case, for translational oscillations along both the X- and Y-axis, and slosh moment data are needed for rotation oscillations about the X, Y, and Z-axes. Likewise, tests using liquids of various densities are required to separate the effects of bladder stiffness from the effects of linear acceleration on the springs K_1 and K_2 (similarly to equation (II.1)). Tests using large-amplitude rotations are needed to define the angle θ and the stiction force between the slider pads and the guiding surface.

III. EXPERIMENTAL PLAN FOR MODEL VERIFICATION

Experimental data were required at a sufficient number of conditions so that numerical values could be evaluated for all parameters postulated in the previous section. The approach for selection of an appropriate set of such sufficient conditions is now described. This approach was selected only after conduct of preliminary experiments which included visual observations and motion picture recording of several different test conditions. These experiments revealed the presence of several types of interface slosh modes, the degree to which each was excited for a given type of excitation, and the degree of nonlinearities present in the response. A similitude analysis of the conditions to be studied is considered. However, use of a true simulation is not intended, nor is actually possible. Instead, a distorted simulation is utilized whereby sufficient information can be obtained to allow complete evaluation of analytical model parameters.

A. Similitude Concepts

The use of a full geometric scale model tank with actual bladder installed was considered feasible. However, it was desirable to employ a simpler simulated propellant, such as water, rather than the actual hydrazine. Furthermore, simulation of testing under other than just 1-g acceleration was highly desirable in order to assure validity of the model over the required range of thrust and zero-g conditions. Therefore, the following similitude concepts were developed to guide the modeling process.

The physical parameters considered important in the problem are as follows:

- F_D = dynamic force loading of liquid tank, MLT^{-2}
- T_D = dynamic torque loading of liquid on tank, ML^2T^{-2}
- K_B = bladder stiffness, MT^{-2}
- ρ_L = liquid density, ML^{-3}
- ρ_U = ullage density, ML^{-3}
- R = tank major radius, L
- a = steady axial acceleration, LT^{-2}
- X_0 = dynamic excitation amplitude, L
- ω = frequency, T^{-1}

With these parameters, the following set of nondimensional parameters can be derived:

$$\left(\frac{F_D}{\rho_l X_0 \omega^2 R^3}, \frac{T_D}{\rho_l X_0 \omega^2 R^4} \right) = F \left(\frac{K_B}{\rho_l a R^2}, \frac{\omega^2 X_0}{a}, \frac{\rho_l}{\rho_u}, \frac{X_0}{R} \right) \quad (\text{III.1})$$

For a full scale tank and actual bladder, with water used in place of hydrazine, these modeling requirements are satisfied by full-scale motions. Experimental results based on this modeling allowed derivation of masses, hinge points, pendulum lengths, and spring constants at $a = 1.0$ g acceleration.

B. Simulation of Variable Gravity

In order to show that masses, hinge points, etc., for the analytical model do not change with the steady axial acceleration, it was desirable to conduct experiments under a simulation of gravity other than 1.0 g. Furthermore, such results were necessary to separate the gravity part of the effective slosh spring constant from the part associated with effective bladder stiffness. Therefore, simulation of variable axial acceleration by liquid density variation in the full-scale tank and actual bladder was considered. This concept was based on the following arguments using parameters equivalent to those in Equation (III.1).

$$1) \quad \frac{K_B/R}{(\rho_l - \rho_u)aR} \approx \frac{\text{steady stiffness stress}}{\text{steady acceleration pressure}}$$

This requires that

$$(\rho_l - \rho_u)_m = (\rho_l - \rho_u)_p \left(\frac{a_p}{a_m} \right), \quad \begin{matrix} m \sim \text{test model} \\ p \sim \text{flight system} \end{matrix} \quad (\text{III.2})$$

$$2) \quad \frac{\rho_l}{(\rho_l - \rho_u)} \left(\frac{\omega^2 X_0}{a} \right) \frac{R}{R} \approx \frac{\text{dynamic liquid pressure}}{\text{steady acceleration pressure}}$$

This requires either that

$$\omega_m^2 = \omega_p^2 \quad (\text{III.3})$$

and

$$(\rho_l X_0)_m = (\rho_l X_0)_p \quad (\text{III.4})$$

or that

$$X_{om} = X_{op} \quad (III.5)$$

and

$$(\rho_l \omega^2)_m = (\rho_l \omega^2)_p \quad (III.6)$$

For the present application Equations (III.3) and (III.4) were selected, based on results of preliminary data (i.e., there was very little change in natural frequency when Equation (III.4) was followed).

$$3) \quad \left(\frac{\rho_l}{\rho_u} \right) \frac{\omega^2 X_o R}{\omega^2 X_o R} \approx \frac{\text{dynamic liquid pressure}}{\text{dynamic ullage pressure}}$$

This requires that

$$(\rho_l / \rho_u)_m = (\rho_l / \rho_u)_p \quad (III.7)$$

$$4) \quad \left(\frac{X_o}{R} \right)^2 \left(\frac{K_B}{\rho_l a R^2} \right) \left(\frac{a}{\omega^2 X_o} \right) = \frac{K_B X_o^2 / R}{\rho_l X_o \omega^2 R^3} \approx \frac{\text{dynamic stiffness force}}{\text{liquid inertia force}}$$

This requires that Equations (III.5) and (III.6) be satisfied simultaneously, or that Equations (III.3), (III.4) and (III.5) be satisfied simultaneously.

$$5) \quad \frac{F_D}{\rho_l X_o \omega^2 R^3} \approx \frac{\text{liquid dynamic force}}{\text{liquid inertia force}}$$

This requires that

$$F_{Dm} = F_{Dp} \quad (III.8)$$

$$6) \quad \frac{T_D}{\rho_l X_o \omega^2 R^4} \approx \frac{\text{liquid dynamic torque}}{\text{liquid inertia torque}}$$

This requires that

$$T_{Dm} = T_{Dp} \quad (III.9)$$

We now consider a series of experiments for simulation of $a > 1.0$ g. It was desirable to simulate hydrazine at $a = 2.5$ g by testing with 2.5 SG zinc bromide solution at $a = 1.0$ g. The results would allow a separation of bladder stiffness and steady acceleration influence on the natural frequencies. Preliminary experiments indicated that Equations (III.8) and (III.9) were appropriate to assure that similar regions of the nonlinear bladder response range would be experienced. Furthermore, it was observed that (III.3) was approximately met by the results, provided that Equation (III.4) also was imposed. However, Equations (III.5) and (III.7) are violated by this approach. Nevertheless, an exact simulation is not necessary, as the analytical model parameters can still be matched to the distorted model results.

The above conditions result in the following prediction parameters convenient for measurement under $a > g$ conditions, with harmonic excitation.

$$\left(\frac{F_D}{X_0 \omega^2 / g} \right)_m = \frac{\rho_{\ell m}}{\rho_{\ell p}} \left(\frac{F_D}{X_0 \omega^2 / g} \right)_p \quad (\text{III.10})$$

and

$$\left(\frac{T_D}{X_0 \omega^2 / Rg} \right)_m = \frac{\rho_{\ell m}}{\rho_{\ell p}} \left(\frac{T_D}{X_0 \omega^2 / Rg} \right)_p \quad (\text{III.11})$$

Equation (III.10) represents the dynamic apparent mass of the fluid under translation, while Equation (III.11) represents the dynamic apparent moment of inertia of the fluid under rotation.

For $a < 1.0$ g acceleration conditions, the above modeling approach can also be used. However, in this case, another type of distorted model simulation is considered. That is

$$\rho_{\ell} > \text{SG of } 1.0, \quad \text{i.e., zinc bromide.}$$

$$\rho_u = \text{SG of } 1.0, \quad \text{i.e., water}$$

With this Equation (III.7) is written as

$$\left(\frac{\rho_{\ell} - \rho_u}{\rho_u} \right)_m = \left(\frac{\rho_{\ell} - \rho_u}{\rho_u} \right)_p \quad (\text{III.12})$$

This condition, as well as most of the others, are violated, so that a true modeling is not considered. However, the results still allow separation of the stiffness and gravity effects on the natural frequencies of the analytical model under approximately simulated low steady acceleration conditions.

In order to determine the validity of this approach, a series of experiments with 2.0 SG zinc bromide solution over water as ullage was planned. The results of these tests were compared with those of water over air at the same fill conditions. With a sufficiently good comparison of results, a second series was planned with 1.25 SG zinc bromide over water at the same fill conditions in order to separate the effects of gravity from the bladder spring constant at low values of axial acceleration a .

Under all test series conditions, measurement of steady state dynamic response under harmonic excitation according to Equations (III.10) and (III.11), as well as certain other static and dynamic measurements to be described below, were considered sufficient for determination of all required model parameters.

IV. EXPERIMENTAL STUDY

A. Description of Apparatus

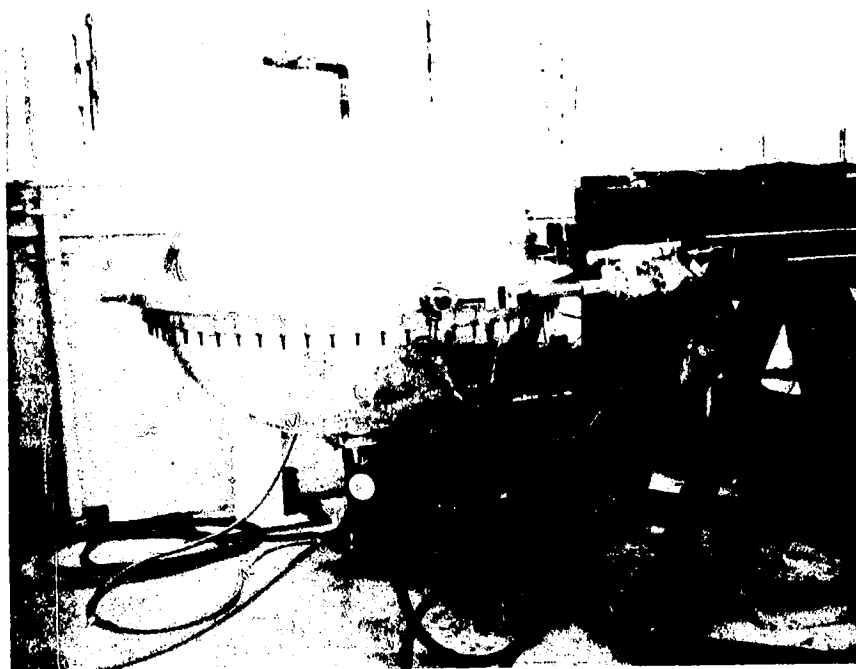
Photographs of the basic apparatus are shown in Figure 2a for X or Y-axis translation, and Z-axis rotation, and in Figure 2b for X or Y-axis rotation. Excitation under the five indicated axes, along with simultaneous measurement of dynamic force and acceleration at the input points were necessary for each configuration test series. For water/air configurations a completely plastic tank was used in order to maximize visual observation. For zinc bromide/air and zinc bromide/water configurations an aluminum top half of the tank was installed in place of the plastic, since zinc bromide was found to cause corrosive deterioration of the plastic.

For X or Y-axis translation, or Z-axis rotation, the tank was supported on the pendulum support shown in Figure 3. Six cables were used for translation, while two hydraulic actuators, attached on each side, were driven in-phase to provide the harmonic excitation. A 3000 lb steel mass was used as reaction support. Acceleration inputs were monitored by maintaining constant displacements from 0 to 4 Hz, and constant acceleration from 4 to approximately 20 Hz. The two inputs were monitored individually, and maintained within ± 1 dB amplitude and within 5° phase angle. This condition was achieved by inputting the identical oscillator signal into both actuators, but providing an amplitude and phase adjust circuit for the input to the second actuator, so that differences in actuator transfer functions could be nulled out. At the same time the input force amplitudes and phases were read and recorded. This procedure was used for most of the test data acquired. Data were hand recorded at discrete frequency settings and later reduced for plotting. For Z-axis rotation only four cables were used on the pendulum support, and the hydraulic actuators were operated to produce input motion of identical amplitude, but exactly out of phase.

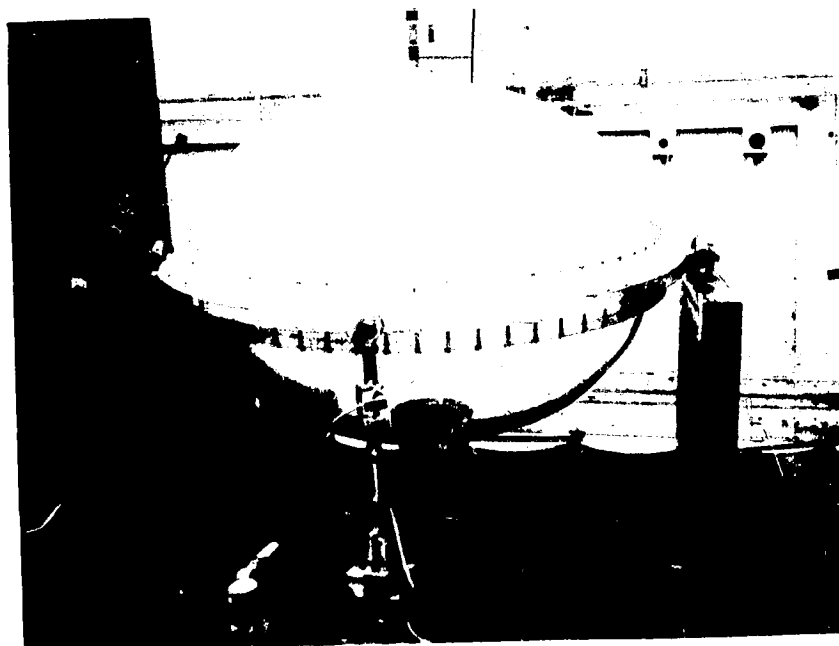
For X or Y-axis rotation the tank was installed in the ball bearing pitch apparatus shown in Figure 2b. In this case only one hydraulic actuator was required. Data were taken under harmonic excitation similar to the other setup.

The nominal TDRS tank was made of 3/8-inch thick lucite plastic. Two ellipsoidal halves were molded and attached to a center aluminum support ring, which also formed the mount for the bladder lip at its circumference. This attachment was essentially the same as in an actual TDRS tank. The center rings were fastened together by sixty bolts and nuts. The rings were also fitted with two standard 3/4-inch machine bolts at opposite sides to allow rotation of the tank about a diametral axis, when supported by only four cables, and the hydraulic actuators detached. This allowed determination of static stability of the bladder surface for various angular orientations of the tank. The nominal interior dimensions were 40 in. major diameter by 30 in. minor axis.

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a. X or Y-Axis Translation & Z-Axis Rotation



b. X or Y-Axis Rotation

FIGURE 2. EXPERIMENTAL APPARATUS ARRANGEMENT

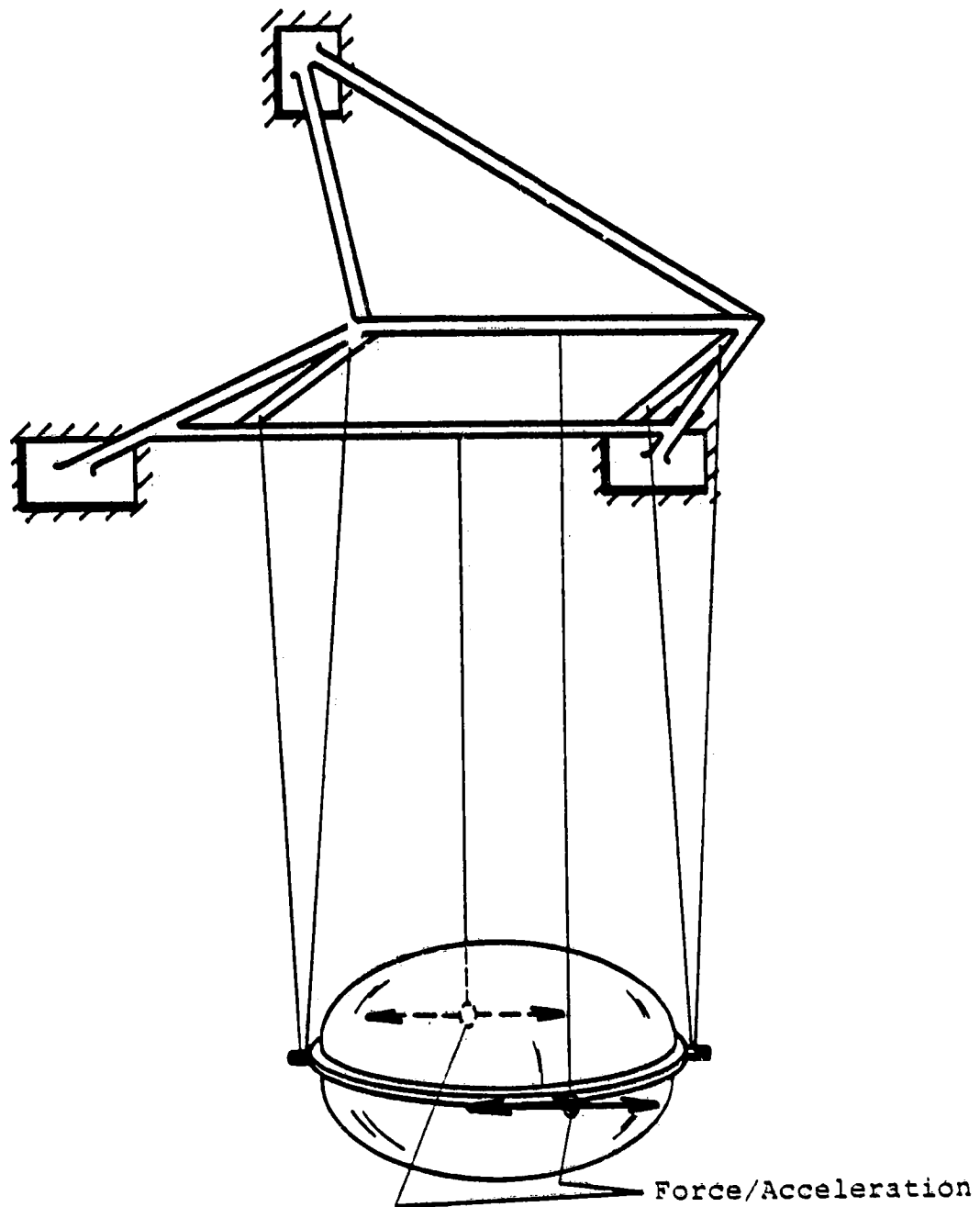


FIGURE 3 PENDULUM SUPPORT

The apparatus for filling the tank to the desired liquid level is shown in Figure 4. A central hoist was used to raise the tank through a four-point cable support. The center wire included a load cell capable of measuring the weight of the tank, filling apparatus, and liquid. Generally, ordinary tap water was added as a simulated propellant on the liquid side of the bladder until the desired weight was achieved. A solution of zinc bromide in water was used for heavier liquids. Two ports with valves attached were included on each aluminum cover plate of the top and bottom. One valve was used for water filling from an ordinary hose, while the other was used to vent air from the side being filled. The ullage side was also fitted with a Bourdon gage for measuring static pressure. Particular care had to be exercised to avoid accumulation of bubbles on the liquid side during filling, or after water had set in the tanks for several hours.

Detail procedures developed for filling the tank necessarily varied with the liquid fill condition which was desired. These details will be given in the individual sections to follow. However, the general ground rules included several cautions. A differential pressure of not over 2 psi was allowed in the ullage. Both sides of the tank were never vented with water in it. The tank could be rotated with no more than 100 pounds of water on the liquid side (this caution was designed to prevent unnecessary strain on the bladder attachments).

B. Liquid Under Ullage - 52 Gallon Configuration

1. Filling Tank

This configuration represents approximately a 1/2-full condition for the upper TDRS tank. In order to fill with water to this condition, both sides of the tank were initially equalized to atmospheric pressure. The tank was then rotated until the liquid side was up and a hose attached. The tank had been hoisted on the weighing apparatus. Initially, about 100 pounds of water were filled into the liquid side. Air was exhausted from the ullage side only if 2 psi were approached. The water pressure was shut off with hose attached and the liquid side was vented to allow all air to escape. Pressure from the ullage was sufficient to force out the air on the liquid side. The air vent was again closed, and with hose still attached, the tank was rotated to the liquid-under position. Subsequently, the water valve was again opened, and the liquid side was filled until the desired liquid weight was achieved. Afterward, the filling hose was removed, the tank was set down onto the pendulum support, and the hoist and weighing apparatus were removed.

After water had set in the tank for several hours, it was found that some dissolved air emerged from solution and caused bubbles under the bladder. This air was removed by first draining most of the water into a separate holding tank through a hose again attached to the

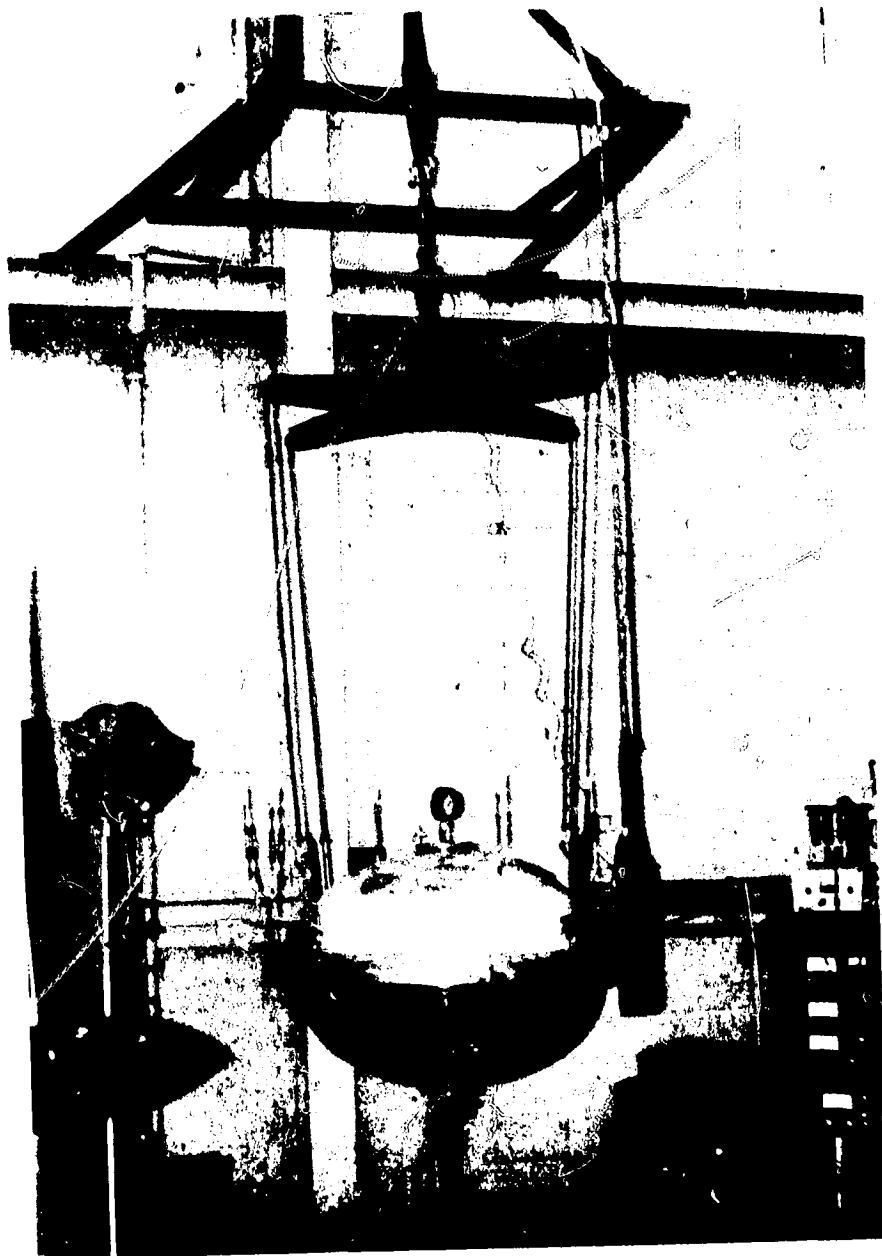


FIGURE 4. TANK FILLING AND WEIGHING APPARATUS

tank. With approximately 100 pounds of water left in the tank, the hose was left attached, but water valve shut off. The tank was rotated until the water was again on top. Opening of the air valve on the water side allowed purging of all remaining air. This valve was then closed and the tank rotated back to the liquid-under position. The holding tank was raised in elevation, the water valve opened, and the water was then returned into the tank by gravity pumping. A little extra water was added to the holding tank to avoid ingestion of air near the end of the filling process. All filling apparatus was removed, and the tank was now ready for experimentation.

2. Description of Bladder Surface

A photograph of the bladder upper surface shape for this configuration is shown in Figure 5. It is not possible to show all aspects of the shape by photograph; so a sketch of the surface in a vertical diametral plane is also given. These data indicate that the shape was fairly symmetrical. This was not always the case after filling to this configuration, for sometimes small local distortions, or folds, in the surface would occur.

In this configuration, the upper access cap of the tank was also removed, and the bladder surface prodded around with a wooden stick. It was found that some distortion of the surface could be imposed. However, upon shaking of the tank, as described in a later section, the surface tended to restore itself to a more or less symmetrical condition.

3. Static Stability of Bladder Surface

Stability of the bladder or its shape under different orientations was studied by two different techniques. Initially, the tank was "bumped" by hand to see how the surface would respond. Generally, it returned to its initial shape with essentially no oscillation. Then, the tank was rotated to several different angular orientations. The position of the bladder for four different angular positions is shown in Figure 6. It can be seen that, essentially, the surface tried to remain horizontal as the liquid would without the bladder present. The essential symmetrical shape is reassumed if returned to the horizontal position from any of the other angular orientations.

4. Steady State Harmonic Response

Response of the bladder surface was photographed with 16 mm film, and the input parameters were measured under harmonic excitation. Figure 7 shows some of the results, while more complete data are given in the appendix for all test runs. Generally, this configuration experienced one interface slosh mode at about 1.3 Hz. Damping of the mode was computed from the free decay record shown in Figure 7. It will

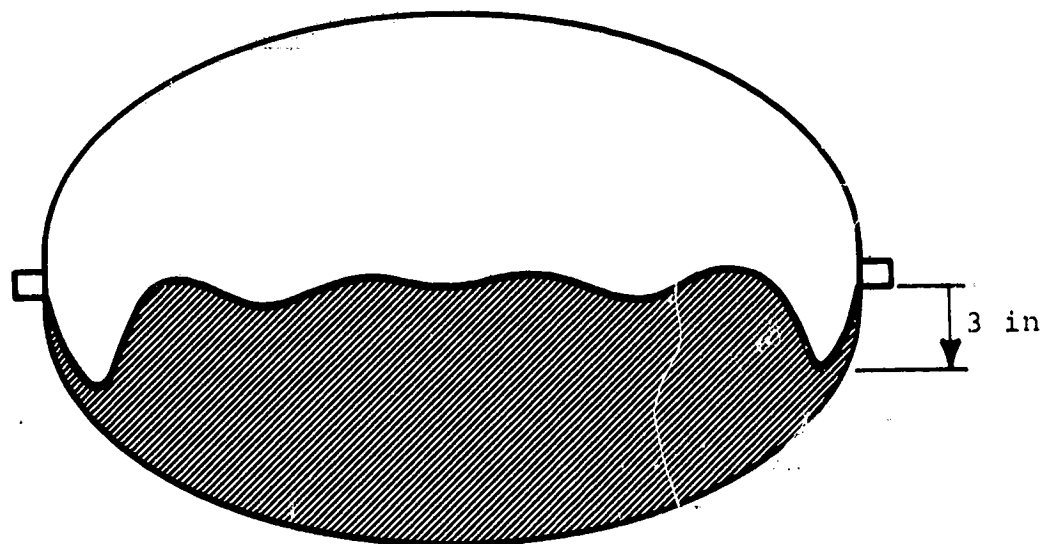
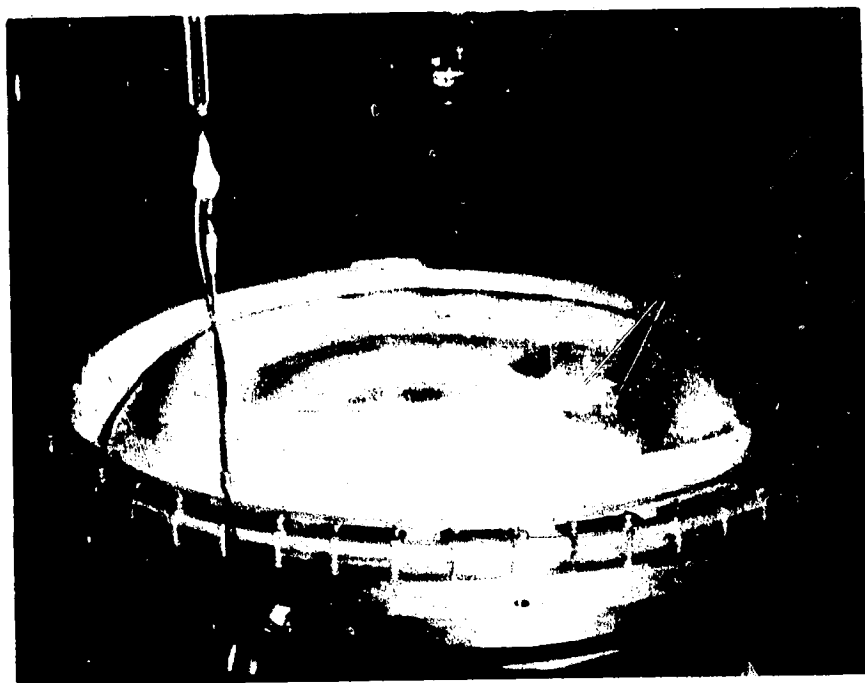


FIGURE 5. BLADDER SURFACE SHAPE FOR 52-GAL.
WATER UNDER ULLAGE

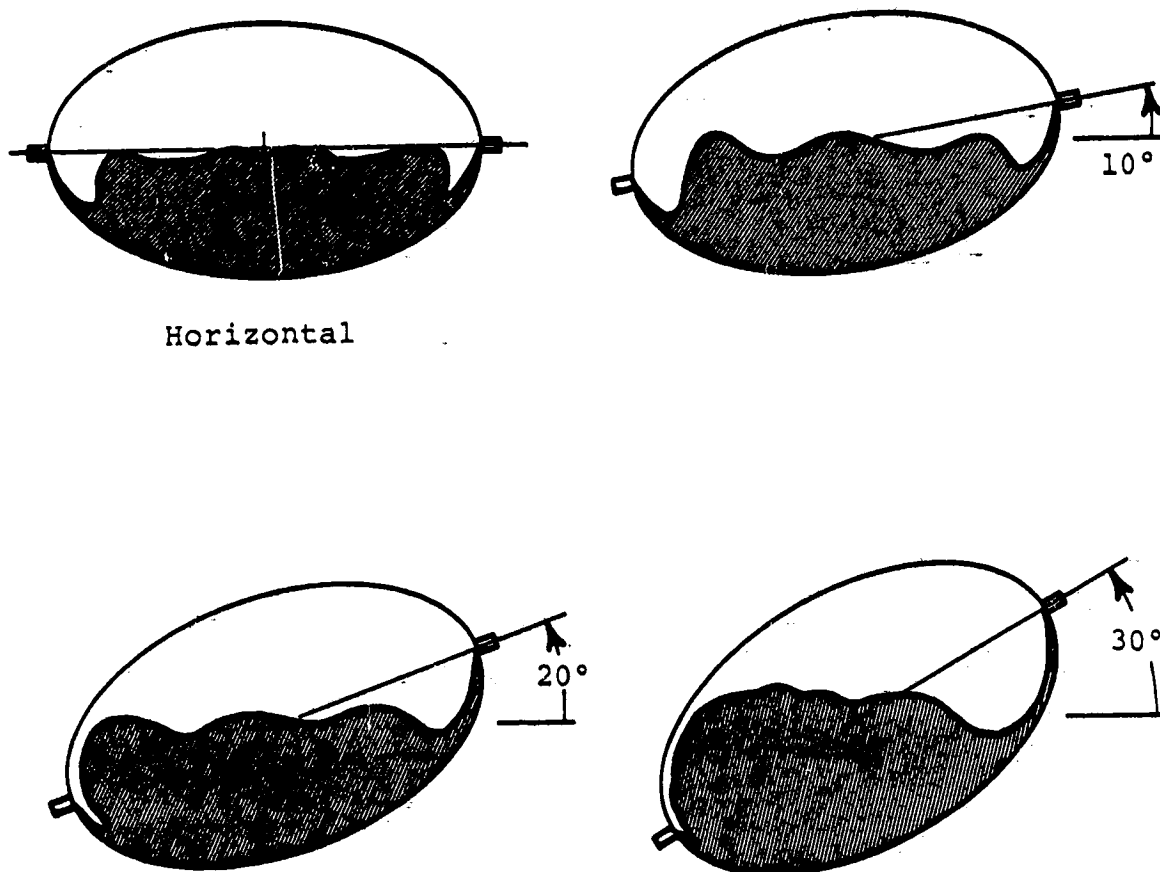
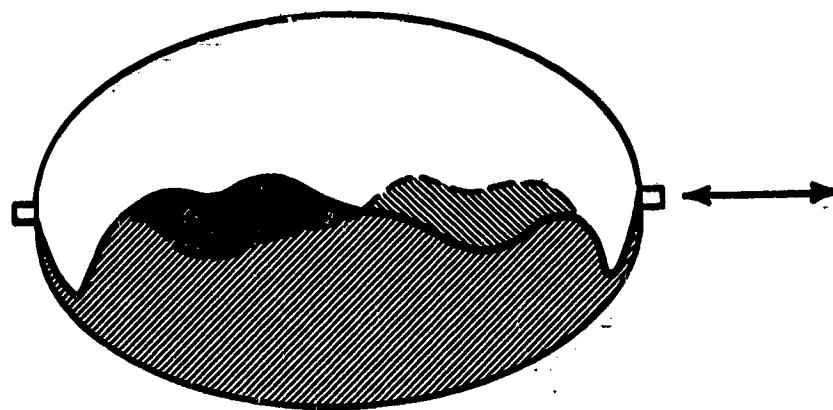


FIGURE 6. STATIC STABILITY OF BLADDER FOR 52 GAL.
WATER-UNDER-ULLAGE CONFIGURATION

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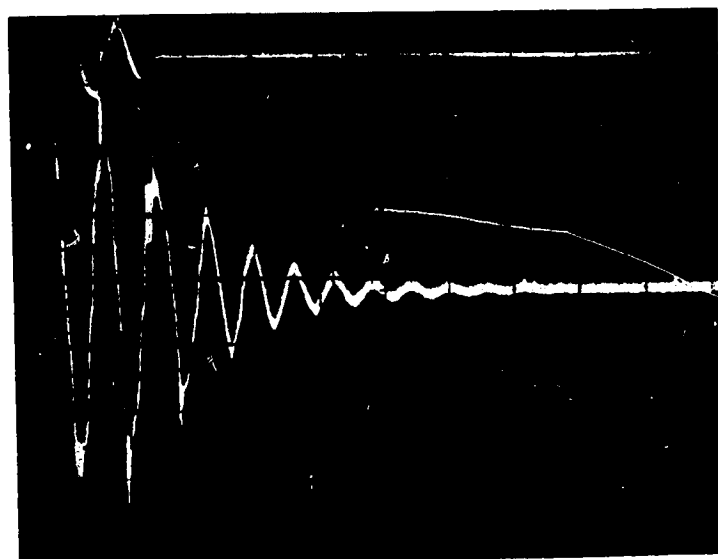


Mode Shape

$$f_1 = 1.3 \text{ Hz}$$

Disp.

Force



Free Decay at 1.3 Hz, $\beta_1 = 0.091$

FIGURE 7. HARMONIC RESPONSE FOR 52-GAL.
WATER-UNDER-ULLAGE CONFIGURATION
X-T EXCITATION

be seen that typical damping values will be quite high, because of the inherent material properties of the bladder. Potential nonlinearities were investigated by acquiring additional data in the vicinity of the resonance. All data plots obtained from the various runs for this and all other configurations are presented in the Appendix. A diagram of the test matrix is given in Table 1, while a summary of results achieved is given in Table 2. Note that vertical (Z-axis) translation data is not included. Complete data under this type excitation were not obtained since preliminary experiments with vertical excitation showed that very little response occurred. Analysis of all this data is given in a later section along with derivation of the parameters for the analytical model.

C. Liquid Under Ullage - 108 Gallon Configuration

1. Filling Tank

This configuration was studied immediately after the one described in the previous section. Therefore, additional water needed to be added to reach 108 gallons. Initially, tap water had been allowed to set in a holding tank for several days to deaerate completely. Then, the plastic TDRS tank with 52 gallons of water at the bottom was raised by the weighing apparatus until all support from the pendulum support was relaxed. With the holding tank at a level below the TDRS tank, a hose was attached to the lower tank water valve. This valve was opened and all air from the hose allowed to vent through the holding tank. Then, the holding tank was hoisted to a level above the TDRS tank, and air-free water flowed into the tank until a water weight of 900 pounds was achieved. All filling apparatus was then removed, and we were ready to proceed with this configuration.

2. Description of Bladder Surface

A photograph and cross-sectional sketch of the bladder surface for this shape are shown in Figure 8. In this case, the surface tended to assume the more or less symmetrical form quite readily. The configuration represents a nearly full condition for the upper TDRS tank.

3. Static Stability of Bladder Surface

Again, the bumping and angular tilt approaches were used to study stability of the surface. No appreciable oscillation followed the bumps applied in the horizontal plane. Results of angular reorientation are shown in Figure 9. As before, essential behavior seems to be that the bladder surface simply tends to remain horizontal. It should also be emphasized that for this case, as well as for the previously described 52-gallon case, if the tank is returned to the horizontal position from any of the angular orientations indicated, the surface simply re-assumes the more or less symmetrical shape.

TABLE 1. SUMMARY OF TEST RUNS CONDUCTED

A. Liquid Under Ullage

| Configuration † | X-T | Y-T | Z-R | X-R | Y-R | Amp. in-pp | Static Stability |
|-------------------------------|-----|-----|-----|-----|-----|---------------|---------------------|
| 52 Gal. Water | A1* | A1 | | | | 0.25 | X |
| 108 Gal. Water | A2 | A2 | | A3 | A3 | 0.25 | X |
| 108 Gal. Water Under Water | | | | A4 | A4 | 0.25 | |

B. Liquid Over Ullage

| Configuration | X-T | Y-T | Z-R | X-R | Y-R | Amp. in-pp | CG Offset | Static Stability | Dynamic Friction |
|---|-----|-----|-----|-----|-----|---------------|--------------|---------------------|---------------------|
| 54 Gal. Water | R** | | R | | R | 0.12 | | | |
| | B1 | B2 | B3 | B4 | B5 | 0.25 | X | X | X |
| | R | | R | | R | 0.50 | | | |
| 2.52 SG 54 Gal. Zn. Br. | B11 | | | R | R | 0.05 | | | |
| | B6 | B7 | B8 | B9 | B10 | 0.10 | X | X | X |
| | B12 | B13 | B14 | R | R | 0.25 | | | |
| 2.52 SG 36 Gal. Zn. Br. | B15 | B16 | B17 | | | 0.25 | | | |
| 2.0 SG 54 Gal. Zn. Br. Over Water | B18 | B19 | B20 | B21 | B22 | 0.25 | X | X | |
| 1.25 SG 54 Gal. Zn. Br. Over Water | | | | B23 | B24 | 1.00 | | | |

* Numbers refer to Figures in the Appendix

** R designates resonance search only

† X-T = X-Axis Translation, Y-T = Y-Axis Translation, Z-R = Z-Axis Rotation, etc.

TABLE 2. SUMMARY OF RESONANCE DATA

A. Liquid Under Ullage

| Configuration | X-T | | Y-T | | Z-R | | X-R | | Y-R | | Amp. | CG Offset |
|----------------------------|-----|---------|-----|---------|-----|---------|-----|---------|-----|---------|-------|-----------|
| | Hz | β | Hz | β | Hz | β | Hz | β | Hz | β | in-pp | in. |
| 52 Gal. Water | 1.3 | 0.091 | 1.3 | 0.091 | | | | | | | 0.25 | |
| 108 Gal. Water | 1.9 | | 1.9 | | | | 2.4 | | 2.4 | | 0.12 | |
| | 1.6 | 0.110 | 1.6 | 0.110 | | | 2.2 | 0.110 | 2.2 | 0.110 | 0.25 | |
| | 1.3 | | 1.3 | | | | 2.2 | | 2.2 | | 0.50 | |
| 108 Gal. Water Under Water | | | | | | | 1.0 | | 1.0 | | 0.25 | |

B. Liquid Over Ullage

| Configuration | X-T | | Y-T | | Z-R | | X-R | | Y-R | | Amp. | CG Offset |
|---|-----|---------|-----|---------|-----|---------|-----|---------|-----|---------|-------|-----------|
| | Hz | β | Hz | β | Hz | β | Hz | β | Hz | β | in-pp | in. |
| 54 Gal. Water | 1.9 | | | | 2.3 | | | | 2.1 | | 0.12 | |
| | 1.6 | 0.116 | | | 2.4 | | 8.0 | | 1.7 | 0.094 | 0.25 | 4.81 |
| | 1.4 | | | | 1.9 | | | | 1.7 | | 0.50 | |
| 2.52 SG 54 Gal. Zn. Br. | 1.6 | 0.068 | | | | | 8.1 | | 1.5 | | 0.50 | |
| | 1.5 | 0.077 | 7.5 | | 1.6 | 0.082 | 7.6 | 0.110 | 1.5 | 0.067 | 0.10 | 5.05 |
| | 1.3 | 0.093 | | | 1.7 | | 7.5 | | 1.4 | | 0.25 | |
| 2.52 SG 36 Gal. Zn. Br. | 0.9 | | | | 1.0 | | | | | | 0.25 | |
| 2.0 SG 54 Gal. Zn. Br. | | | | | 0.8 | | 6.0 | | 0.8 | | 0.25 | 1.53 |
| 1.25 SG 54 Gal. Zn. Br. Over Water | | | | | | | 4.0 | | 0.8 | | 1.00 | 0.57 |

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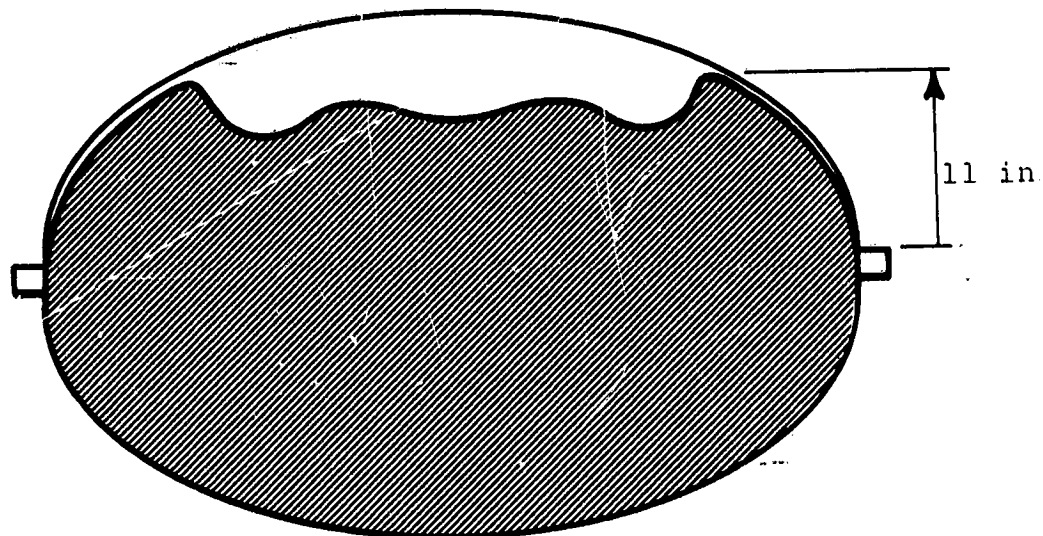
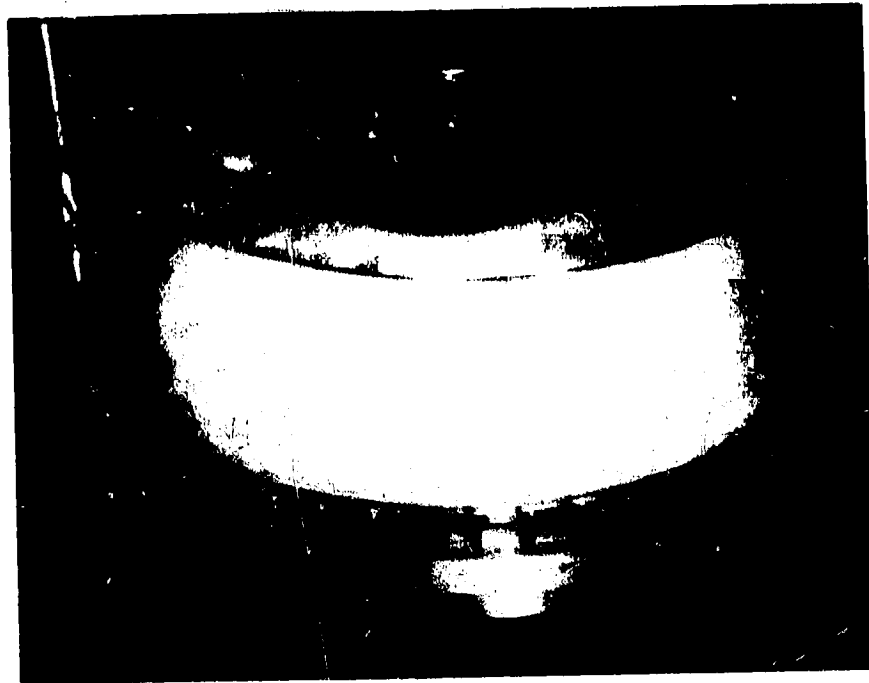


FIGURE 8. BLADDER SURFACE SHAPE FOR
108 GAL. WATER UNDER ULLAGE

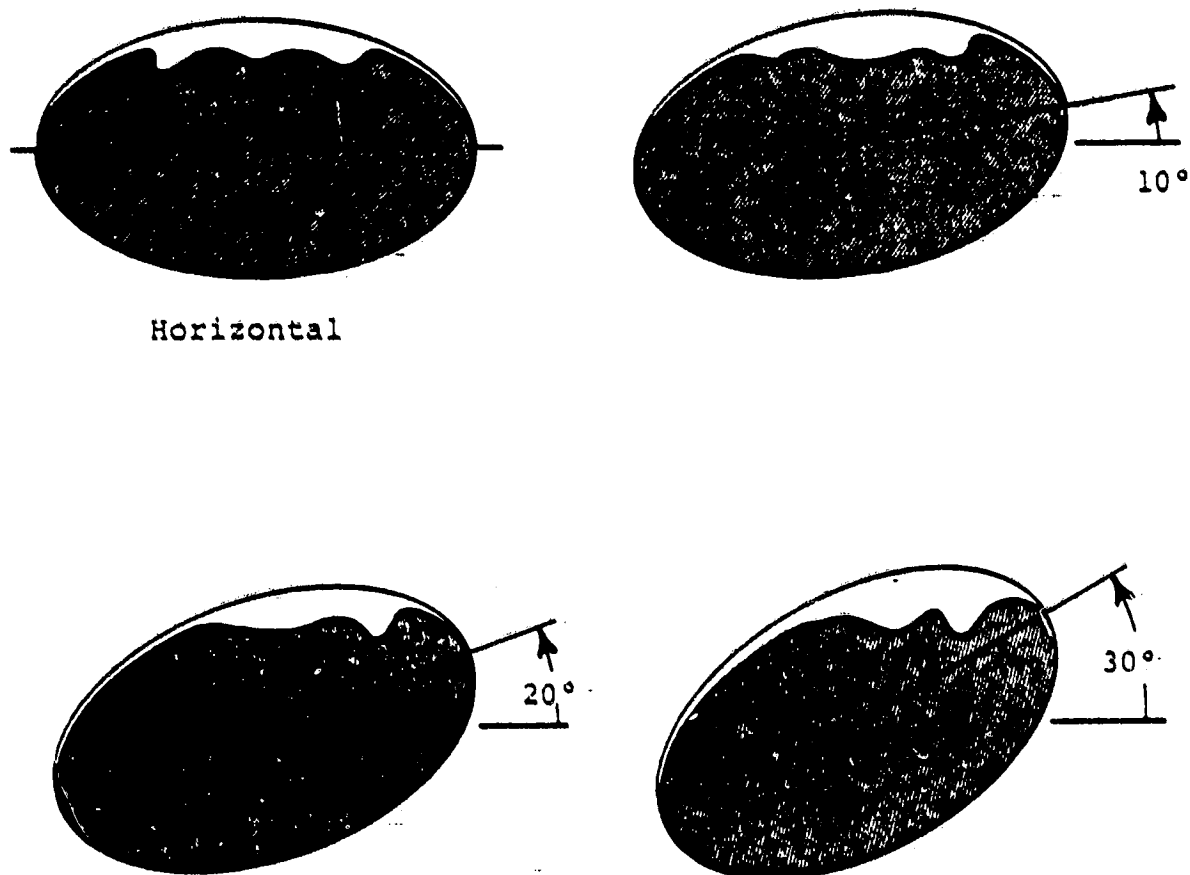


FIGURE 9. STATIC STABILITY OF BLADDER FOR 108-GAL.
WATER-UNDER-ULLAGE CONFIGURATION

4. Steady State Harmonic Response

Movies and input parameters were again acquired for steady state forced response in a frequency range up to 20 Hz. The same input motion amplitudes were used as with the previous configurations, and the resulting input forces were recorded. Some results are shown in Figure 10. In this case, an antisymmetric slosh mode occurs at 1.6 Hz. From its indicated mode shape, it can be seen that motion occurs principally over about 75% of the surface diameter. Free decay resulted in a damping ratio of 0.110. Nonlinearity with input motion amplitude was found to be very similar to that of the previous case.

D. Liquid-Over-Ullage Configuration

1.. Tank Filling Procedure

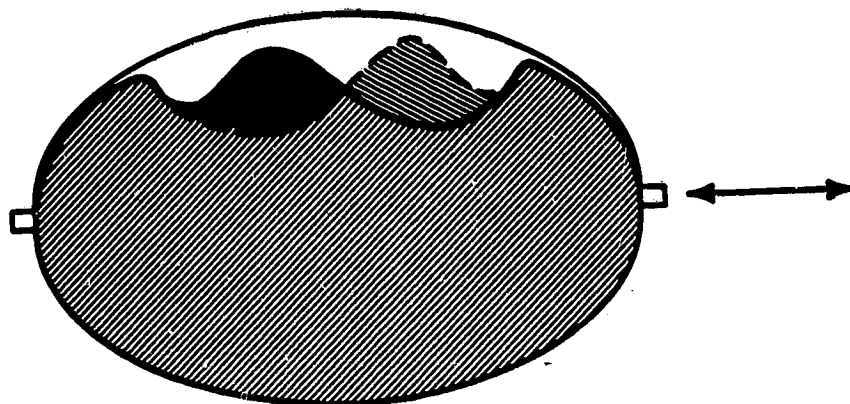
This was the third configuration studied. To obtain the desired fill condition, first, all but about 100 pounds of water was drained by gravity into the holding tank. Then, the TDRS tank was rotated to where the liquid side was on top. The tank was then attached to and hoisted slightly by the weighing apparatus. Thereafter, additional water was drained by gravity from the holding tank into the top of the TDRS tank, until the desired water weight was achieved. Subsequently, the air valve on top was opened until all air bubbles were vented. Finally, the weighing apparatus was removed, so that the TDRS again rested on the pendulum support.

There were several different liquid over ullage configurations for which data runs were performed, as can be seen from Tables 1 and 2. The description given in this section applies principally to the 54 gallon water over air and 2.52 SG zinc bromide over air configurations. However, procedures for the other cases were similar, and data for all runs identified in Table 1 are given in Table 2 and the Appendix.

2. Description of Bladder Surface

The 54-gallon water over air configuration represents about a half-full condition for the lower TDRS tank. A photograph and diametral cross-section sketch are shown in Figure 11. The shape of the surface was essentially symmetrical on each side of this diametral plane. For this case the photograph is somewhat misleading, as light distortion tends to show the bladder wrapped around at the upper right hand side of the tank. This does not occur, but slight wraparound of the bladder does occur at each side (as seen in the foreground of the photo).

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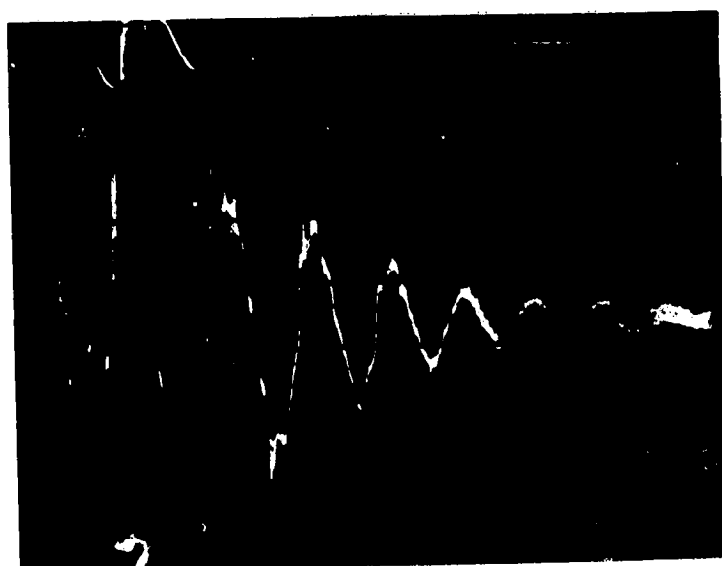


Mode Shape -

$$f_1 = 1.6 \text{ Hz}$$

Disp.

Force



Free Decay at 1.6 Hz, $\beta_1 = 0.110$

FIGURE 10. HARMONIC RESPONSE FOR 108-GAL.
WATER-UNDER-ULLAGE CONFIGURATION
X-T EXCITATION

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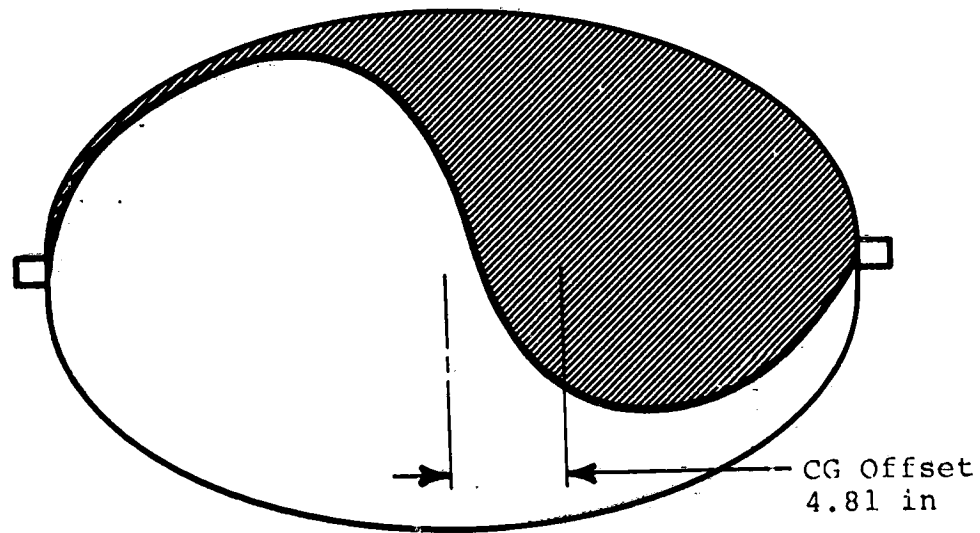
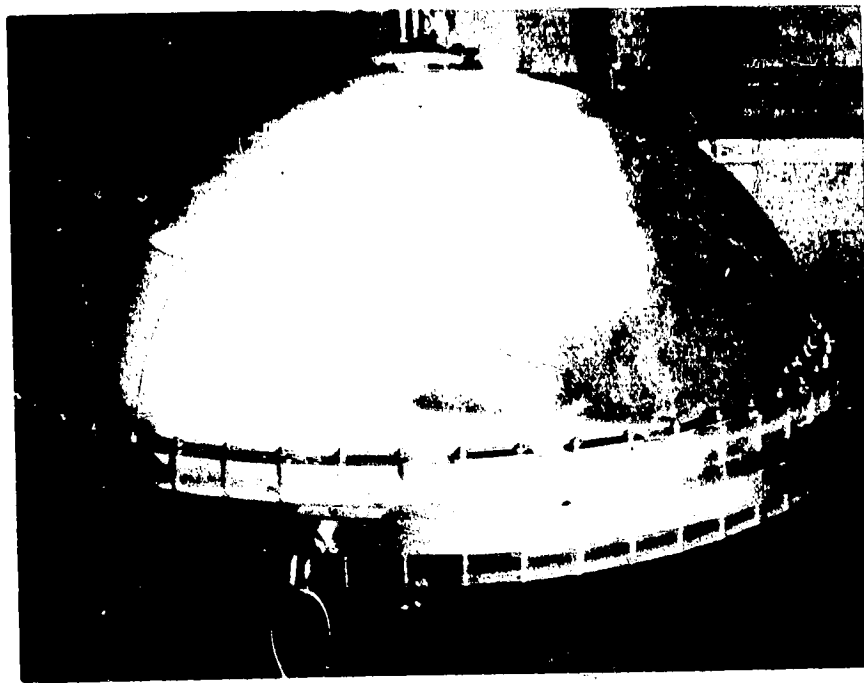


FIGURE 11. BLADDER SURFACE SHAPE FOR
54 GAL. WATER OVER ULLAGE

As indicated in the sketch, the horizontal offset of the liquid center of gravity was 4.81 inches. This was determined by weighing the tank at three support positions and subtracting out the effects of the tank itself. The vertical position of the center of gravity was not determined. Similar offsets for other configurations are given in Table 2.

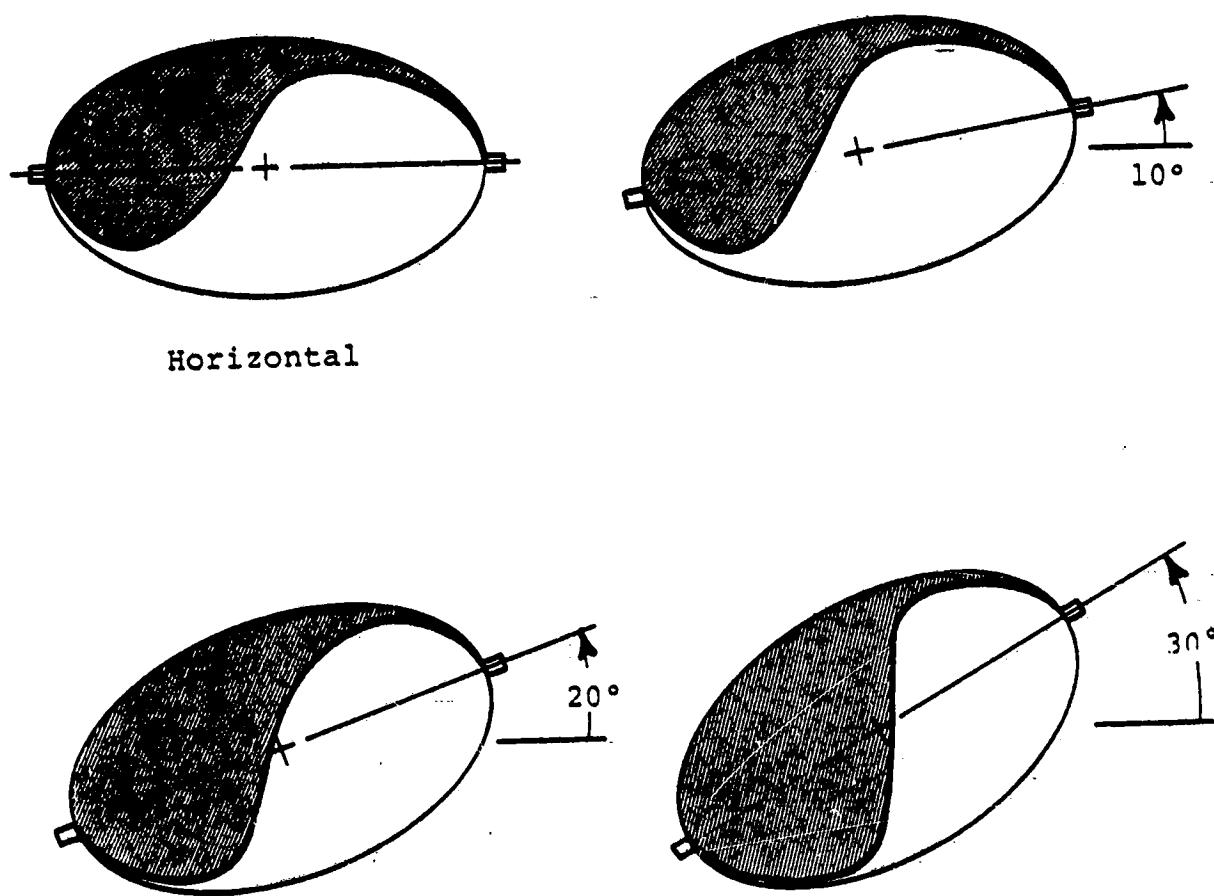
3. Static Stability of Bladder

First, it must be said that there was absolutely no tendency for the liquid to stay on center. To the contrary, the overwhelming tendency was for the liquid to flop into the side position shown, although the exact side location could, of course, be anywhere about the vertical axis. It then became pertinent to consider what would make the liquid move to some other side position.

Initially, the tank was tilted by 10° increments toward the liquid side, in the diametral plane of symmetry. Figure 12 shows the results of this approach with 54 gallons of water. Essentially, the liquid and bladder remained stable and tended to shift more and more to the downward side. Then, the tank was tilted toward the ullage side in the diametral plane of symmetry. Results are shown in Figure 13. Note that at 10° very little occurs. However, as approximately 19° to 20° were approached slowly, flop-over occurred in the plane of symmetry. That is, the liquid flopped directly over the top, with no tendency for it to creep around the side. It is conceivable that the latter might happen if the angular positions were changed extremely slowly. Once the liquid flopped over, the 30° position was essentially the same as that in Figure 12.

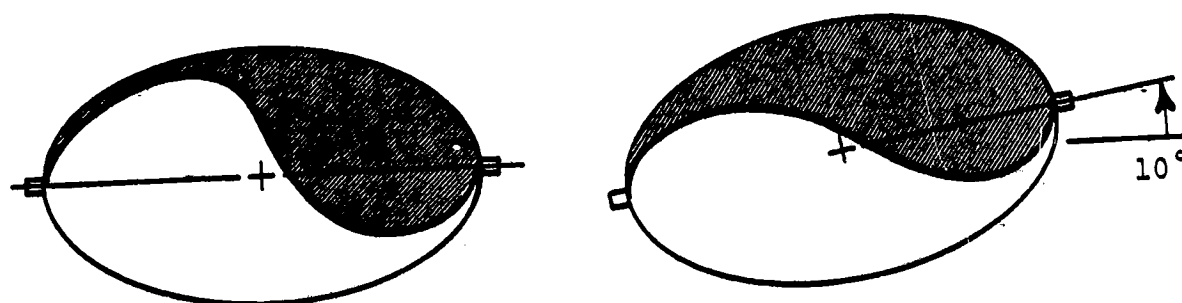
The tank was also tilted in a 90° cross plane. The results are shown in Figure 14. It is necessary to show a top view of the tank to describe what happens. Here it is understood that liquid is present to some extent near the tank wall in the upper half. However, cross hatching is employed to indicate where most of the liquid appears in the top view. One local fold is shown on each side of the tank. The tank is then tilted slowly as indicated in the figure. At 5° very little occurs. At about 10° the liquid and bladder begin to creep around to approximately 45° from its initial position. By tilting to 20° the surface has completely reoriented as indicated. Similar results were obtained for other liquid over ullage configurations, and will be summarized later.

For 54 gallon water, more accurate data were taken for bumping of the tank in the horizontal plane. The electrohydraulic actuator was used to input a half sine transient in the plane of symmetry for the bladder surface. The intent was to see if the bladder could be made to flop over to the other side. Some results are shown in Figure 15 as input displacement and force time histories. It was found that even

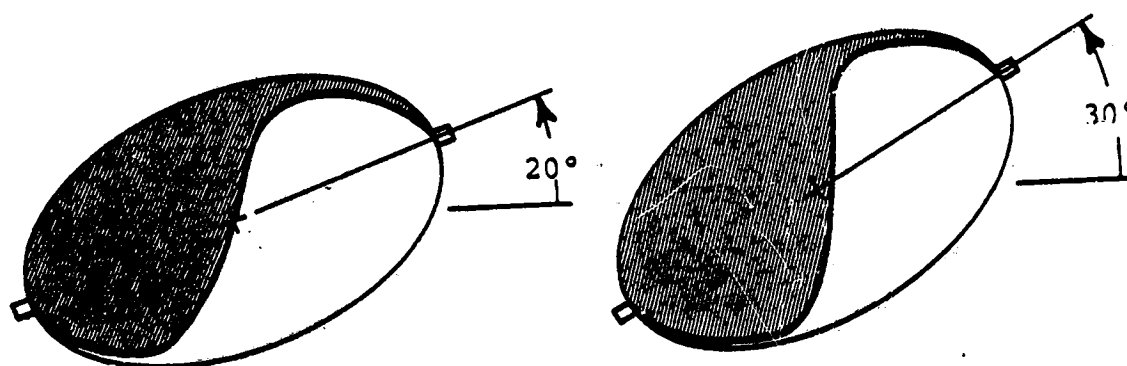


a) Shift toward Liquid Side

FIGURE 12. STATIC STABILITY OF BLADDER FOR 54-GAL.
WATER-OVER-ULLAGE CONFIGURATION



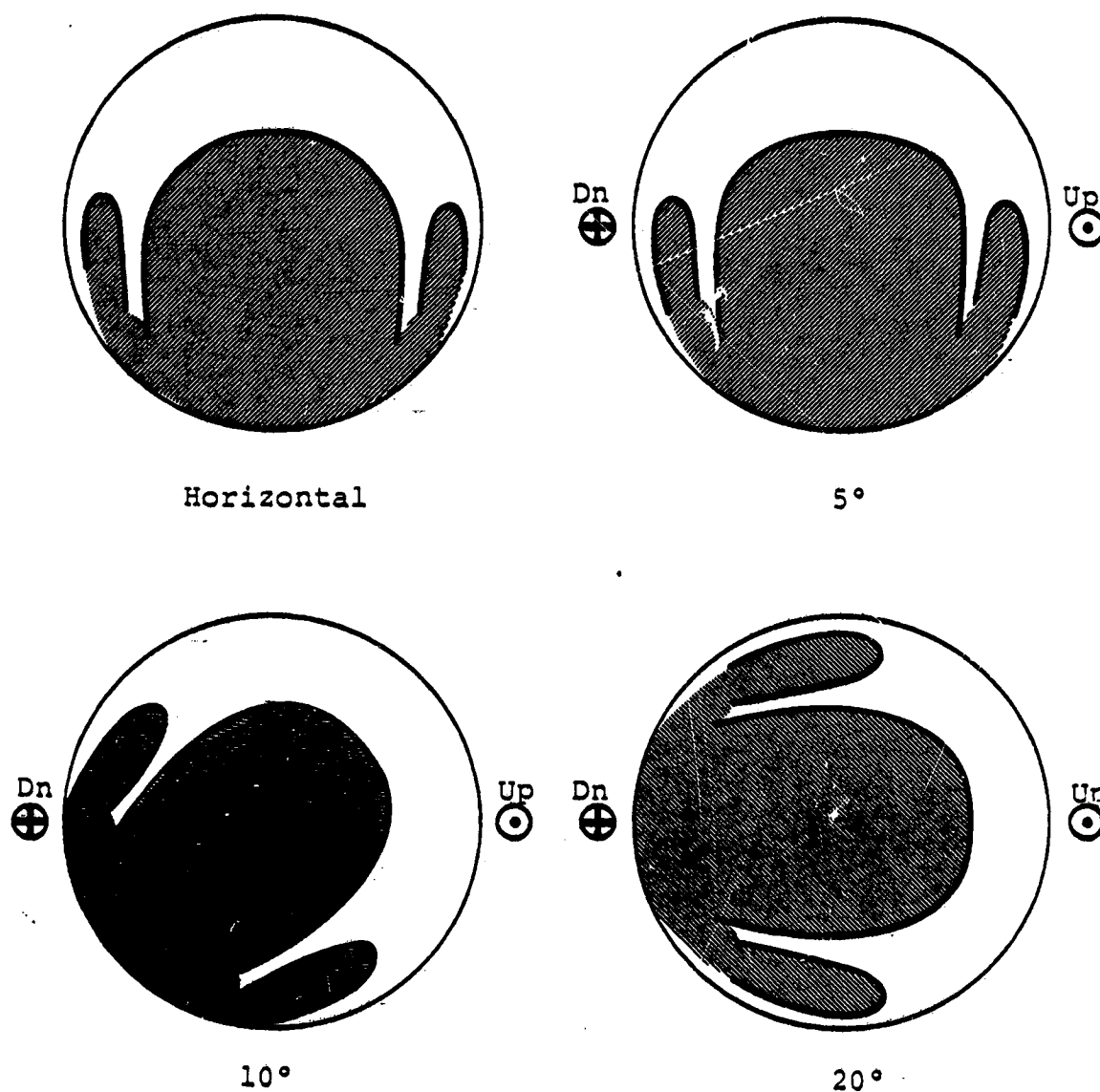
Horizontal



NOTE: Flopover occurs about 19° - 20° .
No tendency for liquid to circle
around tank.

b) Shift toward Ullage Side

FIGURE 13. STATIC STABILITY OF BLADDER FOR 54-GAL.
WATER OVER ULLAGE CONFIGURATION



c) Shift to Cross-Axis Side

FIGURE 14. STATIC STABILITY OF BLADDER FOR 54-GAL.
WATER-OVER-ULLAGE CONFIGURATION.

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Input
Disp. 1.25 in.

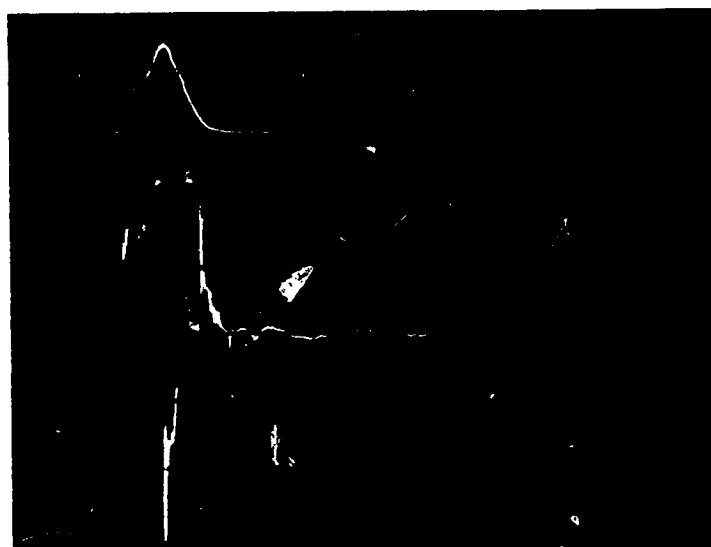
Input
Force 100 lb.



→ 1 sec. ←

Input
Disp. 1.25 in.

Input
Force 100 lb.



→ 1 sec. ←

FIGURE 15. STATIC STABILITY OF BLADDER FOR 54-GAL.
WATER-OVER-ULLAGE CONFIGURATION
Transient Pulse Response

with rather large displacements (either pulling or pushing at the tank), the surface remained essentially unchanged, and in fact, very little oscillation of the surface occurred.

4. Dynamic Friction of Bladder

For the 54-gallon water over air case an additional experiment was performed to help determine the creep or dynamic friction properties of the bladder. The tank configuration was set on the X-Y axis rotation apparatus shown in Figure 2, without the actuator attached. The side of the tank was supported by a hoist instead, in order to be able to apply relatively large angular pitch displacements suddenly. Initially, the bladder bubble was set up so that the tank would rotate about the Y-axis. The tank was then suddenly tilted to a given angular pitch displacement, and the bubble angular position about a vertical axis was observed visually as a function of time. Figure 16 shows some results of this experiment, for three different initial pitch on tilt angles. For any larger tilt angles the bubble shifted to 90° very rapidly. For smaller angles there was very little shift.

5. Steady State Harmonic Response

With the liquid over ullage configuration, it can be seen that shaking along two different principal axes is logical. Figure 17 shows results where the tank was excited along the X-axis in translation. One resonance occurs at 1.6 Hz. The mode shape is easiest described by the indicated top view of the tank. Essentially the mode is antisymmetric where the inclined portion of the bladder surface which is parallel to the excitation is most involved. No such low frequency mode was found for translation excitation along the Y-axis (in the plane of symmetry). However, later evidence with 54 gallons of 2.52 zinc bromide solution over air in Y-axis translation, and with both water and zinc bromide solutions in X-axis pitch, indicates clearly that a highly damped mode appears at about 7.5 to 8 Hz. Some information on this mode is given in Figure 18. By consulting the plotted data in the Appendix it can be seen that this mode causes significant offloading of mass at high frequencies. The existence of this mode was not at all obvious when the experiments were begun, and tended to be highly obscured with the water-over-air configuration.

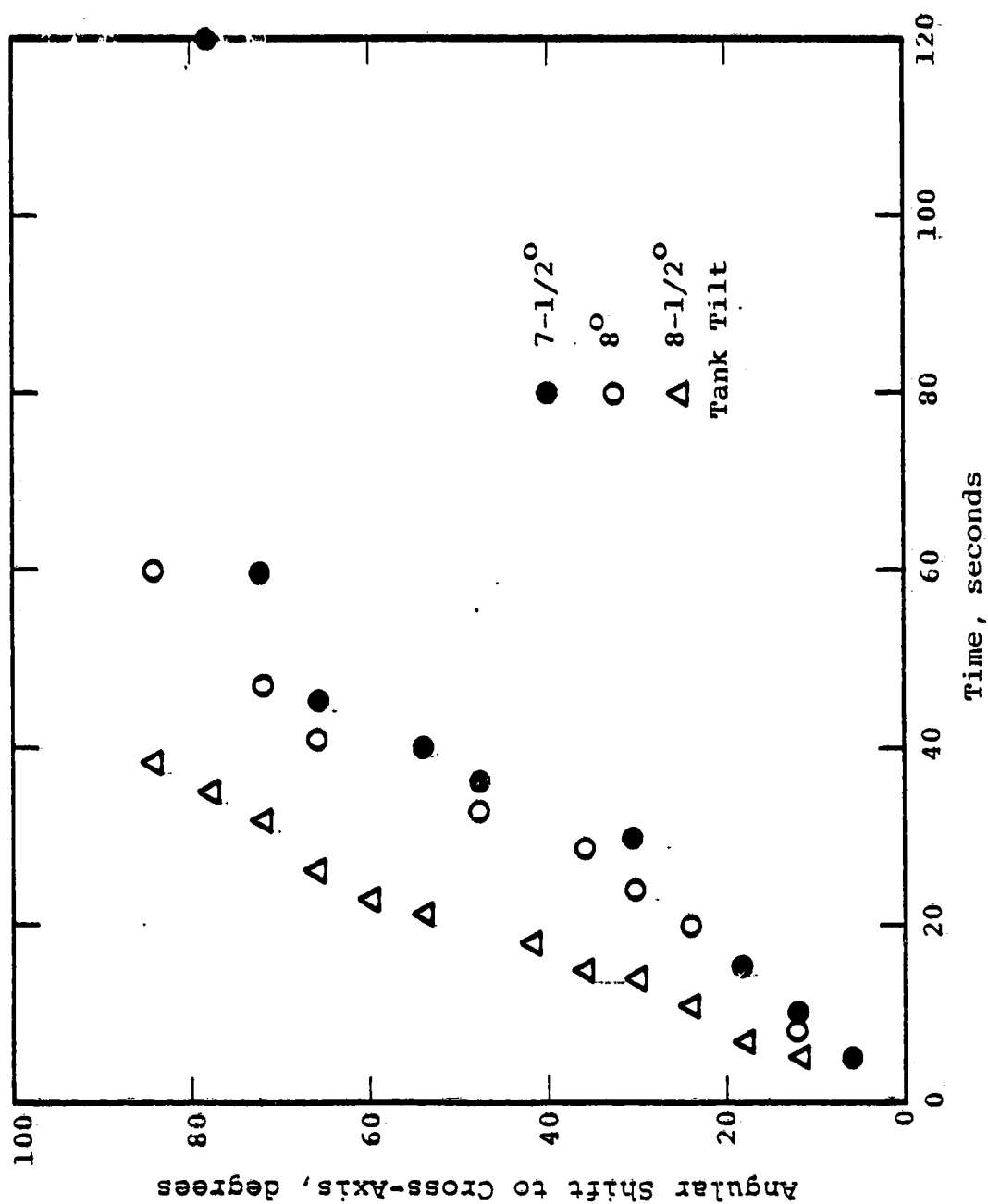
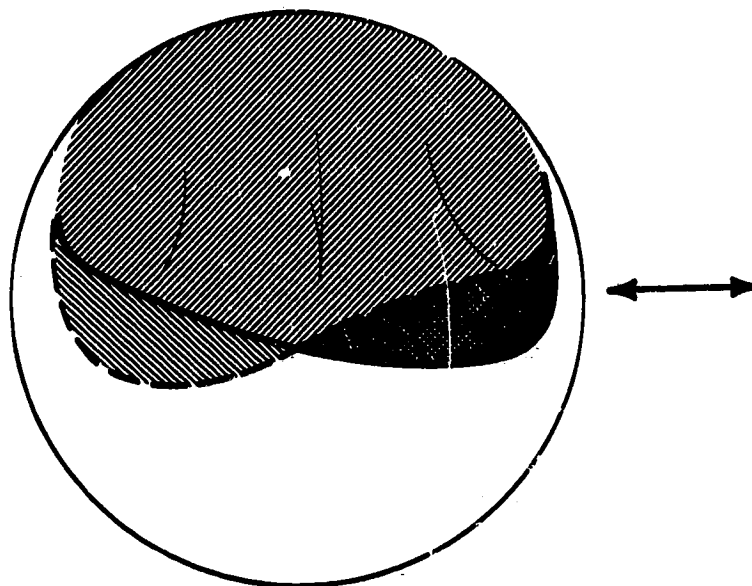


FIGURE 16 DYNAMIC FRICTION OF BLADDER FOR 54-GAL
WATER-OVER-ULLAGE CONFIGURATION

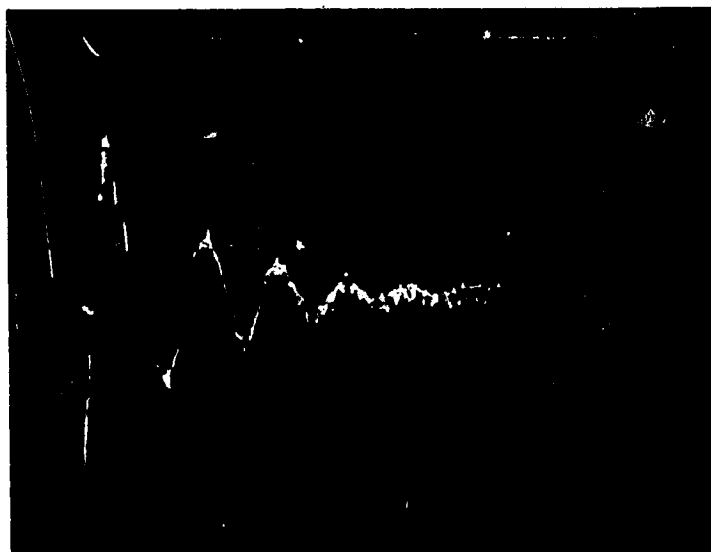
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Mode Shape
 $f_1 = 1.6 \text{ Hz}$

Disp.

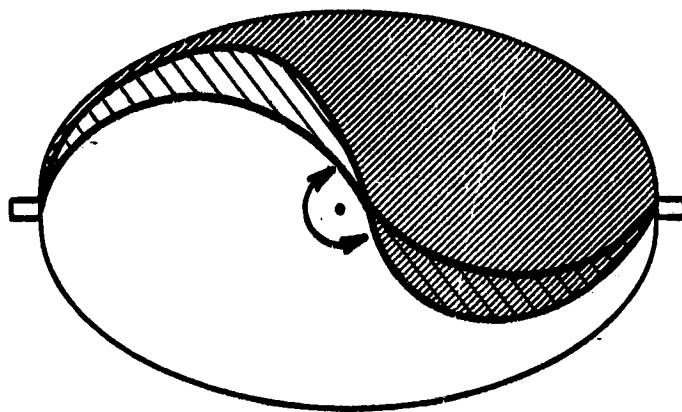
Force



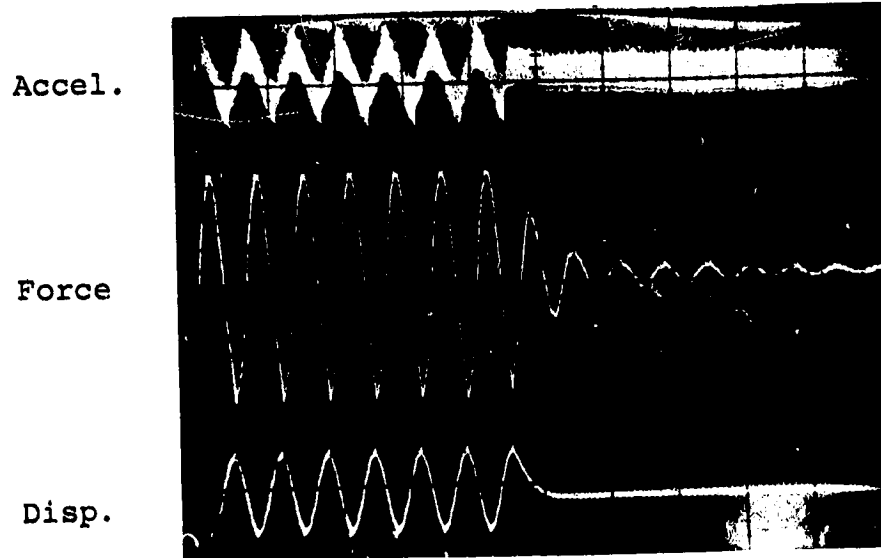
Free Decay at 1.6 Hz, $\beta_1 = 0.116$

FIGURE 17. HARMONIC RESPONSE FOR 54-GAL.
WATER-OVER-ULLAGE CONFIGURATION
X-T EXCITATION

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Mode Shape
 $f_2 = 7.6 \text{ Hz}$



Free Decay at 7.6 Hz, $\beta_2 = 0.110$

FIGURE 18. HARMONIC RESPONSE FOR 54-GAL.
2.52 SG ZINC BROMIDE-OVER-ULLAGE CONFIGURATION
X-R EXCITATION

E. Liquid-over-Liquid Configurations

In section III.B an approach for partial simulation of variable gravity was presented. That approach leads to the concept of a distorted physical model with which measurements are made to verify the adequacy of the postulated analytical model. The distorted model includes the use of zinc bromide mixtures over water as ullage, to simulate partially a low gravity condition. With this concept, 54-gal of 2.0 SG zinc bromide over water was used to simulate water over air at 1.0 g for comparison with data obtained from the 54-gal water over air configuration. Comparison of the results must be made with the analytical model. These details will be included in the Appendix. Subsequently, 54 gal water at 0.25 g was simulated by 54 gal 1.25 SG zinc bromide over water, and 54 gal water at 0 g was simulated with 108 gal water under water. Pertinent results from these tests will also be discussed in the Appendix.

V. DEVELOPMENT OF MODEL PARAMETERS

A. Liquid-Under-Ullage Model

1. Equations of motion. The slosh model for the forward tank was shown previously as Figure 1a. Letting ψ_1 be the angular rotation of the pendulum relative to the angular rotation of the tank, the equation of motion for the pendulum is

$$\ddot{\psi}_1 + \left(\frac{C_\theta}{m_1 L_1^2} \right) \dot{\psi}_1 + \left(\frac{K_\theta + m_1 L_1 a}{m_1 L_1^2} \right) \psi_1 = - \left[\ddot{y}_0 + (L_1 - h_1) \ddot{\phi}_x + a \phi_x \right] / L_1 \quad (V.1)$$

where y_0 is the tank translational excitation along the Y-axis and ϕ_x the tank rotational excitation around the X-axis. The forces and moments predicted by the model, including the contribution of the immobile mass and the empty tank inertia, are

$$F_y = -\omega^2 y_0 e^{i\omega t} \left\{ m_T + m_0 + m_1 + m_1 \Omega^2 / [1 - \Omega^2 + 2 i \beta \Omega] \right\} \\ - \omega^2 \phi_x e^{i\omega t} \left\{ m_0 h_0 + m_1 (L_1 - h_1) \right. \\ \left. + m_1 (L_1 - h_1) [\Omega^2 + a / (L_1 - h_1) \omega_1^2] / [1 - \Omega^2 + 2 i \beta \Omega] \right\} \quad (V.2)$$

and

$$M_x = -\omega^2 y_0 e^{i\omega t} \left\{ m_0 h_0 + m_1 (L_1 - h_1) \right. \\ \left. + m_1 (L_1 - h_1) [\Omega^2 + a / (L_1 - h_1) \omega_1^2] / [1 - \Omega^2 + 2 i \beta \Omega] \right\} \\ - \omega^2 \phi_x e^{i\omega t} \left\{ I_T + I_0 + m_0 h_0^2 + m_1 (L_1 - h_1)^2 \right. \\ \left. + m_1 (L_1 - h_1)^2 [\Omega^2 + a^2 / (L_1 - h_1) \Omega^2 \omega_1^4 + 2a / (L_1 - h_1) \omega_1^2] / \right. \\ \left. [1 - \Omega^2 + 2 i \beta \Omega] \right\} \quad (V.3)$$

In these equations, a simple harmonic excitation $e^{i\omega t}$ has been assumed; also m_T and I_T are the empty tank mass and moment of inertia, $\omega_1 = [(K_\theta + m_1 L_1 a)/m_1 L_1^2]^{1/2}$ is the slosh frequency, $\beta = C_\theta/2 m_1 \omega_1 L_1^2$ is the damping ratio, and $\Omega = \omega/\omega_1$.

2. Empirical determination of parameters. Numerical values of the model parameters have been computed for the two test filling levels: 92.7% full and 44.6% full. The model for the 92.7% filling is described first.

Experimental data available for determining the parameters are given in Figures A2, A3, and A4. The relevant data can be summarized as follows:

Translation excitation, y_0

- resonant frequency ≈ 1.6 hz
- resonant force ≈ 1350 lb per g
- damping ratio ≈ 0.11
- for $\Omega \gg 1$, "rigid body" force ≈ 1050 lb per g

Rotation excitation, ϕ_x

- resonant frequency ≈ 2.2 hz
- resonant moment ≈ 295 in-lb-sec²
- damping ratio ≈ 0.11
- for $\Omega \gg 1$, "rigid body" moment ≈ 200 in-lb-sec²

"Zero-g" rotation excitation, ϕ_x

- resonant frequency ≈ 1.0 hz

The resonant frequencies for rotational and translational excitation differ somewhat, although visual observations during the tests confirmed that the same mode was excited in both cases. The difference is apparently due to nonlinearities in the bladder motion. Since translation excites the mode a little more cleanly than rotation does, the translation frequency is assumed to be the more appropriate estimate.

The weight of the empty tank and test fixtures was determined to be 284 lb, and the moment of inertia was 170 in-lb-sec². The center of mass was calculated to lie 0.99 inches below the center of the tank.

Using equation (V.2) with $m_T + m_0 + m_1 = 1184$ lb (900 lb liquid and 284 lb tank), the calculated best fit to the data requires $m_1 = 78$ lb, $m_0 = 822$ lb, and $\omega_1 = 1.6/0.94 = 1.7$ hz. (With large damping, the resonant frequency is slightly less than the natural frequency.) The predicted high-frequency limiting value of the force exerted on the tank is 1105 lb per g (this is just $m_0 + m_T$), which is fairly close to the measured 1050 lb per g.

Values of K_θ and L_1 individually cannot be determined solely from knowledge of ω_1 . Instead, the "zero-g" simulation must also be used. For that test, the ullage above the bladder was filled with liquid (71 lb). Since there cannot be an unbalanced gravitational force on the liquid for such a configuration, the bladder stiffness K_θ provides the only restoring force that can act on the slosh mass. However, the slosh mass must include the contribution of the oscillating liquid in the ullage. Since the ullage liquid forms essentially a shallow tank, all of it participates in the sloshing [1]; thus, $m_1 = 149$ lb. The two values of rotational resonant frequency can now be used to calculate K_θ : $\omega_1 = 2.2$ hz for $m_1 = 78$ lb and $\omega_1 = 1.0$ hz for $m_1 = 149$ lb. The computed K_θ is 175.8 in-lb. (The small difference between resonant frequencies and natural frequencies are neglected in this calculation.) With $K_\theta = 175.8$ in-lb, the translation natural frequency of 1.7 hz requires a pendulum length $L_1 = 4.9$ inches, a value that agrees well with that for a corresponding unbladdered tank [1].

Using equation (V.3) with $I_T + I_0 + m_1 h_1^2 = 200$ in-lb-sec² (the high-frequency limiting moment), the resonant moment data are best correlated by a value of $L_1 - h_1 = 9.8$ inches. That is, $h_1 = -4.9$ inches; in comparison to an unbladdered tank, the line of action of the slosh mass is shifted downward somewhat. Preservation of the center of mass location requires that $h_0 = 0.15$ inches; the immobile mass is above the slosh mass. Finally, since $I_T + I_0 + m_1 h_1^2 = 200$ in-lb-sec², the moment of inertia I_0 of the immobile mass can be calculated to be 29.9 in-lb-sec². A summary of all these parameters is given in Table 3.a.

The 44.6% filling level was of somewhat less interest. Only a translation excitation test was conducted; the relevant data from Figure A1 are:

Translation excitation, y_0

- resonant frequency ≈ 1.3 hz
- resonant force ≈ 1150 lb per g
- damping ratio ≈ 0.091
- for $\Omega \gg 1$, "rigid body" force ≈ 550 lb per g.

TABLE 3. SLOSH MODEL PARAMETERS FOR FORWARD TANK

3.a 92.7% Filling (900 lb of hydrazine)

$$m_1 = 78 \text{ lb}$$

$$m_0 = 822 \text{ lb}$$

$$L_1 = 4.9 \text{ inches}$$

$$K_\theta = 175.8 \text{ in-lb}$$

$$C_\theta = 11.39 \text{ in-lb-sec}$$

$$I_0 = 29.9 \text{ in-lb-sec}^2$$

$$h_1 = -4.9 \text{ inches}$$

$$h_0 = 0.15 \text{ inches}$$

3.b 44.6% Filling (433 lb of hydrazine)

$$m_1 = 135 \text{ lb}$$

$$m_0 = 298 \text{ lb}$$

$$L_1 = 6.4 \text{ inches}^*$$

$$K_\theta = 175.8 \text{ in-lb}^*$$

$$C_\theta = 22.26 \text{ in-lb-sec}^*$$

$$I_0 = 14.4 \text{ in-lb-sec}^{2*}$$

$$h_1 = 0^*$$

$$h_0 = 6.2^*$$

*. Best estimate

Using these data, it turns out that the best correlation is given by $m_1 = 135$ lb, $m_0 = 298$ lb, and $\omega_1 = 1.36$ hz. The predicted high-frequency limiting force is 582 lb, in good agreement with the test. Compared to the 92.7% filling results, the value of m_1 agrees with the known result that the slosh mass in a spheroidal tank has its maximum value near a half-full level [1]. The remaining parameters must be estimated, since other test data are not available. The bladder stiffness K_0 is assumed to be equal to the 92.7% filling value, namely 175.8 in-lb. Since $\omega_1 = 1.36$ hz, the required pendulum length is thus 6.4 inches. For an unbladder tank of the same shape and filling level, the pendulum hinge point $h_1 \approx 0$ [1]; this value is assumed to hold for the bladder tank as well. The immobile mass must therefore be 6.2 inches below the centerline in order to preserve the center of mass location. The moment of inertia I_0 of the immobile mass is assumed to be in the ratio of the filling levels with respect to the 92.7% level; that is, $I_0 = 29.9 (.446/.927) = 14.4$ in-lb-sec². All these values are summarized in Table 3.b.

B. Liquid-Over-Ullage Model

1. Equations of motion. The model representing the sloshing dynamics, as contrasted to static stability characteristics, is considered first. Figure 1b shows the model. Letting X_1 be the displacement of the slosh mass in the X-direction (i.e., the low frequency mode) and y_1 similarly for the y-direction (i.e., the high frequency mode), the equations of motion of the slosh mass are:

$$\ddot{X}_1 + 2\beta_1 \omega_1 \dot{X}_1 + \omega_1^2 X_1 = -\ddot{X}_0 - h_1 \ddot{\phi}_y + r_1 \ddot{\phi}_z \quad (V.4)$$

and

$$\ddot{y}_1 + 2\beta_2 \omega_2 \dot{y}_1 + \omega_2^2 y_1 = -\ddot{y}_0 + h_1 \ddot{\phi}_x \quad (V.5)$$

Here, $\beta_1 = C_1/2m_1\omega_1$, $\beta_2 = C_2/2m_1\omega_2$, $\omega_1 = (K_1/m_1)^{1/2}$, and $\omega_2 = (K_2/m_1)^{1/2}$. By solving these equations for simple harmonic equations, the forces and moments can be found to be:

$$\begin{aligned} F_x = & -\omega X_0 e^{i\omega t} \left\{ m_T + m_0 + m_1 \Omega_1^2 / [1 - \Omega_1^2 + 2i\beta_1 \Omega_1] \right\} \\ & -\omega^2 \phi_y e^{i\omega t} \left\{ m_1 h_1 - m_0 h_0 + m_1 h_1 (\Omega_1^2 + a/h_1 \omega_1^2) [1 - \Omega_1^2 + 2i\beta_1 \Omega_1] \right\} \\ & +\omega^2 \phi_z e^{i\omega t} \left\{ m_0 r_0 + m_1 r_1 + m_1 r_1 (\Omega_1^2 + a/r_1 \omega_1^2) [1 - \Omega_1^2 + 2i\beta_1 \Omega_1] \right\} \end{aligned} \quad (V.6)$$

$$F_y = -\omega^2 y_o e^{i\omega t} \left\{ m_T + m_o + m_1 + m_1 \Omega_2^2 / [1 - \Omega_2^2 + 2i\beta_2 \Omega_2] \right\} \\ + \omega^2 \phi_x e^{i\omega t} \left\{ m_1 h_1 - m_o h_o + m_1 h_1 (\Omega_2^2 + a/h_1 \omega_2^2) / [1 - \Omega_2^2 + 2i\beta_2 \Omega_2] \right\} \quad (V.7)$$

$$M_x = -\omega^2 y_o e^{i\omega t} \left\{ m_1 h_1 - m_o h_o + m_1 h_1 (\Omega_2^2 + a/h_1 \omega_2^2) / [1 - \Omega_2^2 + 2i\beta_2 \Omega_2] \right\} \\ + \omega^2 \phi_x e^{i\omega t} \left\{ I_{Tx} + I_{Ox} + I_x + m_o (h_o^2 + r_o^2) + m_1 (h_1^2 + r_1^2) \right. \\ \left. + m_1 (h_1^2 + r_1^2) [\Omega_2^2 + a^2/H_1^2 \Omega_2^2 \omega_2^4 + 2a/H_1 \omega_2^2] / [1 - \Omega_2^2 + 2i\beta_2 \Omega_2] \right\} \quad (V.8)$$

$$M_y = -\omega^2 x_o e^{i\omega t} \left\{ m_1 h_1 - m_o h_o + m_1 h_1 (\Omega_1^2 + a/h_1 \omega_1^2) / [1 - \Omega_1^2 + 2i\beta_1 \Omega_1] \right\} \\ - \omega^2 \phi_y e^{i\omega t} \left\{ I_{Ty} + I_{Oy} + m_o h_o^2 + m_1 h_1^2 \right. \\ \left. + m_1 h_1^2 (\Omega_1^2 + a^2/h_1 \omega_1^2 \omega_1^4 + 2a/h_1 \omega_1^2) / [1 - \Omega_1^2 + 2i\beta_1 \Omega_1] \right\} \quad (V.9)$$

and

$$M_z = \omega^2 x_o e^{i\omega t} \left\{ m_o r_o + m_1 r_1 + m_1 r_1 (\Omega_1^2 + a/r_1 \omega_1^2) / [1 - \Omega_1^2 + 2i\beta_1 \Omega_1] \right\} \\ - \omega^2 \phi_z e^{i\omega t} \left\{ I_{Tz} + I_{Oz} + m_o r_o^2 + m_1 r_1^2 \right. \\ \left. + m_1 r_1^2 (\Omega_1^2 + a^2/r_1 \omega_1^2 \omega_1^4 + 2a/r_1 \omega_1^2) / [1 - \Omega_1^2 + 2i\beta_1 \Omega_1] \right\} \quad (V.10)$$

Here $\Omega_1 = \omega/\omega_1$, $\Omega_2 = \omega/\omega_2$, and $H_1 = (h_1^2 + r_1^2)^{1/2}$. The empty tank inertia properties are m_T , $I_{Tx} = I_{Ty}$, and I_{Tz} .

2. Empirical determination of model parameters. Most of the slosh parameters can be determined from the baseline tests that used water as the test liquid. (The tests that employed other liquids are needed primarily to sort out the effects of bladder stiffness and to

confirm the values derived from the baseline tests.) The relevant data for the 46.3% full tank are given in Figures B1 through B5.

For the tests that excited the low-frequency mode, the required data are as follows:

X-axis translational excitation

- resonant frequency ≈ 1.6 hz
- resonant force ≈ 960 lb per g
- damping ratio ≈ 0.116
- for $\Omega_1 \gg 1$, "rigid body" force ≈ 450 lb per g

y-axis rotational excitation

- resonant frequency ≈ 1.7 hz
- resonant moment ≈ 525 in-lb-sec
- damping ratio ≈ 0.094
- for $\Omega_1 \gg 1$, "rigid body" moment ≈ 215 in-lb-sec

z-axis rotational excitation

- resonant frequency ≈ 2.4 hz
- resonant moment ≈ 340 in-lb-sec
- damping ratio (undetermined)
- for $\Omega_1 \gg 1$, "rigid body" moment ≈ 300 in-lb-sec

For these tests, the empty plastic tank properties are: $m_T = 284$ lb, $I_{Tx} = I_{Ty} = 170$ in-lb-sec², and $I_{Tz} = 250$ in-lb-sec². The center of mass of the liquid is at $y = 4.81$ inches (measured) and $z = 1.4$ inches (estimated). There is again some differences evident in the resonant frequencies; the value selected for the model is 1.6 hz, for reasons similar to those given in Section V. A. Likewise, an average value of damping ratio equal to 0.11 is used.

Using equation (V.6) to correlate the X-axis translation data yields the best estimates of $m_1 = 144$ lb, $m_0 = 306$ lb, and $\omega_1 = 1.7$ hz. The spring constant K_1 is therefore 42.5 lb/in and $C_1 = 0.876$ in-lb-sec. Since it contains contributions from both bladder stiffness and effective

gravity, K_1 can vary with the thrust level; the variation will be determined later. The damping constant may also vary. With these values of the parameters, the slosh force and phase data are predicted fairly well throughout the frequency range. However, the high-frequency limiting force is predicted to be 590 lb per g, which is somewhat larger than the measured value of 450 lb per g.

Since the high-frequency limiting moment per unit angular acceleration for excitation about the y-axis is 215 in-lb-sec² (from the tests) and in the model is $I_{Ty} + I_{oy} + m_o h_o^2$, equation (V.9) can be used to compute the value of h_1 that best correlates the resonant moment; it is $h_1 = 8.5$ inches. To preserve the center of mass location requires, then, that $h_o = 1.9$ inches (below the centerline). The compiled value of I_{oy} works out to be 42 in-lb-sec².

The z-axis rotation data can be correlated with equation (V.10) in order to predict the best value of r_1 . Realizing that $I_{Tz} + I_{oz} + m_o r_o^2$ must be 300 in-lb-sec², the model predicts that $r_1 = 2.0$ inches. Thus, r_o must be 1.1 inches in order to preserve the center of mass location, and I_{oz} must be 49.0 in-lb-sec².

The high-frequency mode was not prominent in the water tests. The only reliable data from these tests are the high-frequency limiting force of 700 lb per g for y-axis translational excitation and the high-frequency limiting moment of 200 in-lb-sec² for x-axis rotational excitation. The model predicts a limiting force of 590 lb per g, the same as for x-axis translational excitation. The high-frequency limiting moment would imply that $I_{ox} + I_{ix} = 14$ in-lb-sec², but this is perhaps beyond the limits of accuracy of the data. The zinc bromide tests must be used to predict the other parameters of the high-frequency slosh mode. It will be shown later that these tests can be correlated with the model derived from the water tests merely by increasing the masses and moments of inertia in proportion to the liquid density ratio of 2.52. In particular, $m_1 = 363$ lb and $m_o = 771$ lb. The relevant data for the high-frequency mode from Figures B.6 and B.9 are as follows.

y-axis translational excitation ($Z_n B_r$)

- resonant frequency ≈ 7.5 hz
- resonant force ≈ 1450 lb per g
- damping ratio (undetermined)
- for $\Omega_2 \gg 1$, "rigid body" force (data unreliable)

x-axis rotational excitation ($Z_n B_r$)

- resonant frequency ≈ 7.5 hz
- resonant moment ≈ 730 in-lb-sec
- damping ratio (undetermined)
- for $\Omega_2 \gg 1$, "rigid body" moment ≈ 150 in-lb-sec²,

For this tank, $m_T = 234$ lb, $I_{Tx} = I_{Ty} = 170$ in-lb-sec², and $I_{Tz} = 216$ in-lb-sec².

Equation (V.8) can be used to correlate the resonance curve of the measured moments, which is the more clearly defined resonance of the two relevant tests. The only undetermined parameters in this equation are, however, β_2 and $I_{Tx} + I_{Ty}$. The best value of β_2 is computed to be 0.076, which is somewhat less than that of the low-frequency mode. The computed values of $I_{Tx} + I_{Ty}$ is about zero. The model predicts reasonably well the entire resonance curve above about 4 hz for this x-axis rotational excitation. However, the resonant force peak of the y-axis translational test data is considerably over-predicted. The conclusion is that a single slosh mass cannot be made to fit accurately both the low-frequency and the high-frequency modal data. Since the high-frequency mode appears to be of a more obscure nature, a single slosh mass model is retained nonetheless for the sake of simplicity.

It will be demonstrated later that the dependency of K_1 on the inherent bladder stiffness and on the effective gravity is $K_1 = 9.54 + 32.96(a/g)$, in units of lb/in. Assuming the same proportions hold for K_2 , which must have a total value of 2086 lb/in in order to predict $\omega_2 = 7.5$ hz, its dependence on stiffness and effective gravity is

$$K_2 = 214.8 + 742.5(a/g) \quad \text{lb/in.} \quad (\text{V.11})$$

where g is the standard gravitational acceleration. It will also be shown later that the damping constant should depend on the bladder stiffness in the same way as K_1 does; thus since $C_2 = 6.67$ for the zinc bromide test, the general correlation is:

$$C_2 = 0.0032 [214.8 + 742.5 (a/g)] \quad \text{lb-sec/in} \quad (\text{V.12})$$

Most of the tests employing zinc bromide as the simulated propellant, as well as the tests for which the ullage was filled with liquid, were designed to confirm the values of the model parameters derived above and to determine the dependency of K_1 on the steady acceleration a . The tests for which zinc bromide was the simulated propellant

will be discussed first. Zinc bromide has a specific gravity of 2.52, so the static load supported by the bladder is the same as if the liquid were water and the effective gravity had a value of 2.52 g's. The deformed shape of the bladder was, however, virtually the same as when water was used, and the mode shapes of the sloshing also appeared to be similar. These observations, along with the discussion of the similitude analysis presented in Section III, allowed the conclusion to be drawn that the slosh dynamics for the zinc bromide tests should be correlated by the model derived from the water tests, except of course all masses and inertias should be increased by 2.52. The effective value of K_1 should also be changed, in a way to be determined by the tests, since the relative contributions to K_1 of stiffness and effective gravity have changed. The data available to confirm these suppositions are given in Figures B6, B8, and B10, which are summarized as follows.

x-axis translational acceleration

- resonant frequency ≈ 1.5 hz
- resonant force ≈ 2260 lb per g
- damping ratio ≈ 0.077
- "rigid body" force ≈ 600 lb per g

y-axis rotational acceleration

- resonant frequency ≈ 1.5 hz
- resonant moment ≈ 810 in-lb-sec²
- damping ratio ≈ 0.067
- "rigid body" moment ≈ 200 in-lb-sec²

z-axis rotational acceleration

- resonant frequency ≈ 1.6 hz
- resonant moment ≈ 705 in-lb-sec²
- damping ratio ≈ 0.082
- "rigid body" moment ≈ 215 in-lb-sec²

In fact, the model does correlate these data fairly well, when the test data are re-plotted against non-dimensional frequency Ω_1 in order to remove the difference in ω_1 between the water and zinc bromide tests. The measured resonant slosh forces are predicted with practically no error, and the resonant slosh moments are predicted to within $\pm 10\%$. These close

comparisons help confirm the simulation method. Further, the change in ω_1 from 1.70 hz to 1.58 hz between the water and the zinc bromide tests permitted the stiffness effect of the bladder on K_1 to be computed. As discussed earlier, the result of this computation is:

$$K_1 = 9.54 + 32.96 (a/g) \quad \text{lb/in} \quad (\text{V.13})$$

The decrease in damping ratio from about 0.11 to 0.08 is consistent with the idea that most of the damping arises from bladder viscoelasticity. Ideally, the damping constant, then, should depend on stiffness and effective gravity in the same way as K_1 . The computed C_1 values for the water and zinc bromide tests are, however, in the ratio of 1.7 while the K_1 ratio is 2.2. The discrepancy may be caused partly by difficulties experienced in evaluating the damping ratio experimentally. Considering these facts, it is recommended that C_1 be taken as proportional to K_1 , with the constant of proportionality chosen to equalize the predicted error in the two computed values of C_1 . Thus:

$$C_1 = 0.018 [9.54 + 32.96 (a/g)] \quad (\text{V.14})$$

(The predicted damping ratios are now 0.096 and 0.089 rather than 0.11 and 0.08.)

The tests for which the ullage was filled with liquid were designed primarily to prove that equation (V.13) remains valid for $a < g$, in order that the sloshing as $a \rightarrow 0$ might be predicted with some confidence. These tests did confirm equation (V.13). The rationalization of the model and the tests is, however, not straightforward because the mass of liquid in the ullage must be taken into account. For that reason, the details of the comparisons are presented in Appendix C.

The static stability parameters of the model will now be determined. Both the water and the zinc bromide tests showed that the propellant would suddenly rotate "over the top" of the tank to the opposite side when the tilt of the x-axis exceeded 20° . Thus, the inclination, θ , of the inverted pendulum is 20° . Because $r_1 = 2.0$ inches, geometry requires that $L_1 \sin 20^\circ = 5.85$ inches; likewise the pendulum hinge must be at $h_1 - L_1 \cos 20^\circ = 3.0$ inches (above the center of the tank).

For large rotations about the y-axis, the liquid would slowly shift around the tank after some critical angle was exceeded. For water, this angle was about 8° , and for zinc bromide about 7° . The nearly equal values of the angles indicate that the "stiction" force on the slider pads might be modeled as a dry friction with a constant coefficient of friction. Nonetheless, it is also feasible to account for the observed

variation; the two tests are correlated exactly by

$$\begin{aligned} F_s &= 4.24 + 0.325 m_1 a \sin 20^\circ \\ &= 4.24 + 16.01 (a/g) \text{ lb} \end{aligned} \quad (V.15)$$

(The coefficient of friction is 0.325 and the inherent bladder restraint is 4.24 lb.) After breaking away, the liquid would rotate around nearly 90° , to a new equilibrium position, as was shown previously in Figure 16. During this motion, the average retarding force required to dissipate the initial potential energy of the slosh mass is computed to be about 9% of the weight of m_1 . This corresponds to a dynamic friction coefficient of about 0.266 (neglecting the bladder stiffness). The computed time to reach the new position is only in fair agreement with the observed time of about 60 seconds.

Table 4 summarizes all the parameter values of the slosh model for the aft tank.

TABLE 4. MODEL PARAMETERS FOR AFT TANK FOR 46.3% FILLING

$$m_1 = 144 \text{ lb}$$

$$m_0 = 306 \text{ lb}$$

$$r_1 = 2.0 \text{ inches}$$

$$r_0 = 1.1 \text{ inches}$$

$$h_1 = 8.5 \text{ inches}$$

$$h_0 = 1.9 \text{ inches}$$

$$K_1 = 9.54 + 32.96 (a/g) \text{ lb/in}$$

$$C_1 = 0.018 [9.54 + 32.96 (a/g)] \text{ lb-sec/in}$$

$$K_2 = 214.8 + 742.5 (a/g) \text{ lb/in}^*$$

$$C_2 = 0.0032 [214.8 + 742.5 (a/g)] \text{ lb-sec/in}^*$$

$$I_{ox} + I_{1x} = 0$$

$$I_{oy} = 42 \text{ in-lb-sec}^2$$

$$I_{oz} = 49 \text{ in-lb-sec}^2$$

$$L_1 = 5.85 \text{ inches}$$

$$\theta = 20^\circ$$

$$F_s = 4.24 + 16.0 (a/g) \text{ lb (stiction force)}$$

*

K_2 and C_2 represent constants associated with the high frequency (7-8 Hz)² mode. Time and cost did not permit further testing to allow separation of the mass and gravity effects. Therefore, these effects were arbitrarily assumed to be similar to those associated with the lower frequency mode.

VI. CONCLUSIONS

The postulated analytical models appear to provide a reasonable prediction of virtually all observed liquid/interface static and dynamic behavior. In several cases the need for both amplitude and phase data were necessary to allow a good determination of a resonance under steady state, since the high damping characteristics of the bladder tended to make the responses obscure. The dynamic rotational amplitude data also contained errors below about 0.75 Hz. This error resulted from friction in the rotational apparatus bearings. However, no effect on the final results occurred because of this source of error.

REFERENCES

1. Abramson, H.N., editor, The Dynamic Behavior of Liquids in Moving Containers, NASA SP-106, National Aeronautics and Space Administration, 1966 (2nd Printing available from Southwest Research Institute, San Antonio, Texas).
2. Ross, R.G., Jr., and Womack, J.R., "Slosh Testing of a Spherical Mercury Propellant Tank with Positive-Expulsion Diaphragm," Tech. Memo. 33-632, Jet Propulsion Laboratory, Pasadena, CA, July 15, 1973.

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APPENDIX A

STEADY STATE HARMONIC RESPONSE DATA FOR
LIQUID-UNDER-ULLAGE (FORWARD TANK) CONFIGURATIONS

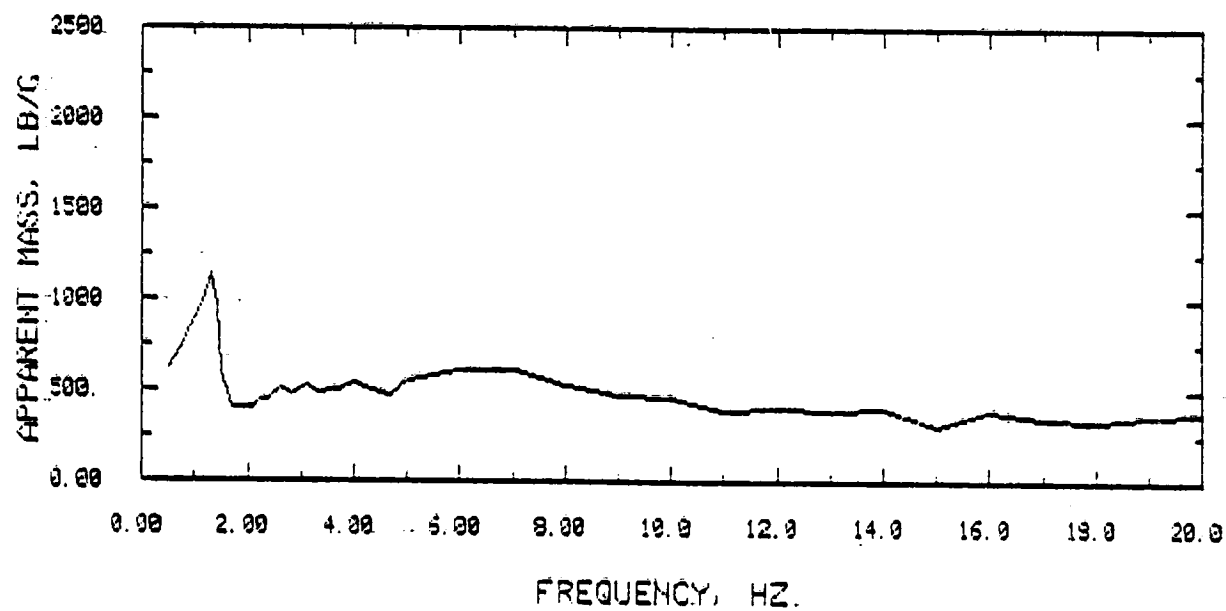
PLOT FIGURE : A1
 CONFIGURATION : LIQUID UNDER AIR ULLAGE
 FILL / LIQUID : 52 GAL. / WATER
 EXCITATION AMPLITUDE : 0.25 INCH P-P
 EXCITATION AXIS-TYPE : X OR Y - TRANSLATION

FREQUENCY- APPARENT MASS
 HZ LB/G

| | |
|--------|----------|
| 0.499 | 617.000 |
| 0.694 | 718.000 |
| 0.822 | 796.000 |
| 1.000 | 885.000 |
| 1.152 | 985.000 |
| 1.287 | 1137.000 |
| 1.406 | 953.000 |
| 1.497 | 584.000 |
| 1.700 | 400.000 |
| 1.821 | 394.000 |
| 1.961 | 390.000 |
| 2.083 | 399.000 |
| 2.222 | 436.000 |
| 2.387 | 439.000 |
| 2.611 | 513.000 |
| 2.809 | 468.000 |
| 3.106 | 518.000 |
| 3.289 | 480.000 |
| 3.676 | 498.000 |
| 4.000 | 529.000 |
| 4.310 | 497.000 |
| 4.651 | 462.000 |
| 5.000 | 554.000 |
| 6.000 | 605.000 |
| 7.000 | 600.000 |
| 8.000 | 515.000 |
| 9.000 | 465.000 |
| 10.000 | 445.000 |
| 11.000 | 380.000 |
| 12.000 | 390.000 |
| 13.000 | 385.000 |
| 14.000 | 390.000 |
| 15.000 | 305.000 |
| 16.000 | 380.000 |
| 17.000 | 340.000 |
| 18.000 | 320.000 |
| 19.000 | 350.000 |
| 20.000 | 370.000 |

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PLOT FIGURE : A1
CONFIGURATION : LIQUID UNDER AIR ULLAGE
FILL / LIQUID : 52 GAL. / WATER
EXCITATION AMPLITUDE : 0.25 INCH P-P
EXCITATION AXIS-TYPE : X OR Y - TRANSLATION

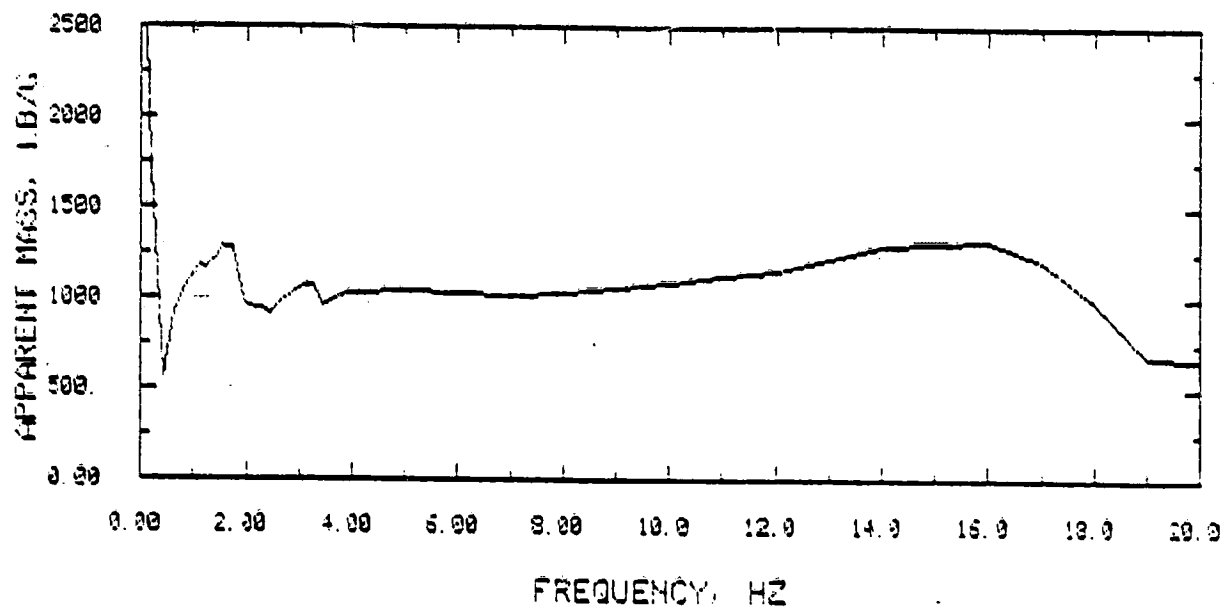


PLOT FIGURE : A2
 CONFIGURATION : LIQUID UNDER AIR ULLAGE
 FILL / LIQUID : 108 GAL. /--WATER
 EXCITATION AMPLITUDE : 0.25 INCH P-P
 EXCITATION AXIS-TYPE : X OR Y - TRANSLATION

FREQUENCY APPARENT MASS ...
 HZ LB/G

| | |
|--------|----------|
| 0.102 | 9415.000 |
| 0.203 | 1702.000 |
| 0.416 | 542.000 |
| 0.608 | 920.000 |
| 0.821 | 1044.000 |
| 1.009 | 1152.000 |
| 1.112 | 1170.000 |
| 1.218 | 1160.000 |
| 1.311 | 1184.000 |
| 1.401 | 1216.000 |
| 1.524 | 1279.000 |
| 1.623 | 1276.000 |
| 1.721 | 1274.000 |
| 1.831 | 1143.000 |
| 1.938 | 979.000 |
| 2.028 | 951.000 |
| 2.242 | 934.000 |
| 2.445 | 916.000 |
| 2.646 | 978.000 |
| 2.841 | 1022.000 |
| 3.040 | 1058.000 |
| 3.247 | 1057.000 |
| 3.448 | 954.000 |
| 3.650 | 998.000 |
| 3.861 | 1023.000 |
| 4.000 | 1027.000 |
| 5.000 | 1040.000 |
| 6.000 | 1015.000 |
| 7.000 | 1005.000 |
| 8.000 | 1020.000 |
| 9.000 | 1050.000 |
| 10.000 | 1075.000 |
| 11.000 | 1125.000 |
| 12.000 | 1150.000 |
| 13.000 | 1220.000 |
| 14.000 | 1280.000 |
| 15.000 | 1300.000 |
| 16.000 | 1310.000 |
| 17.000 | 1200.000 |
| 18.000 | 980.000 |
| 19.000 | 680.000 |
| 20.000 | 665.000 |

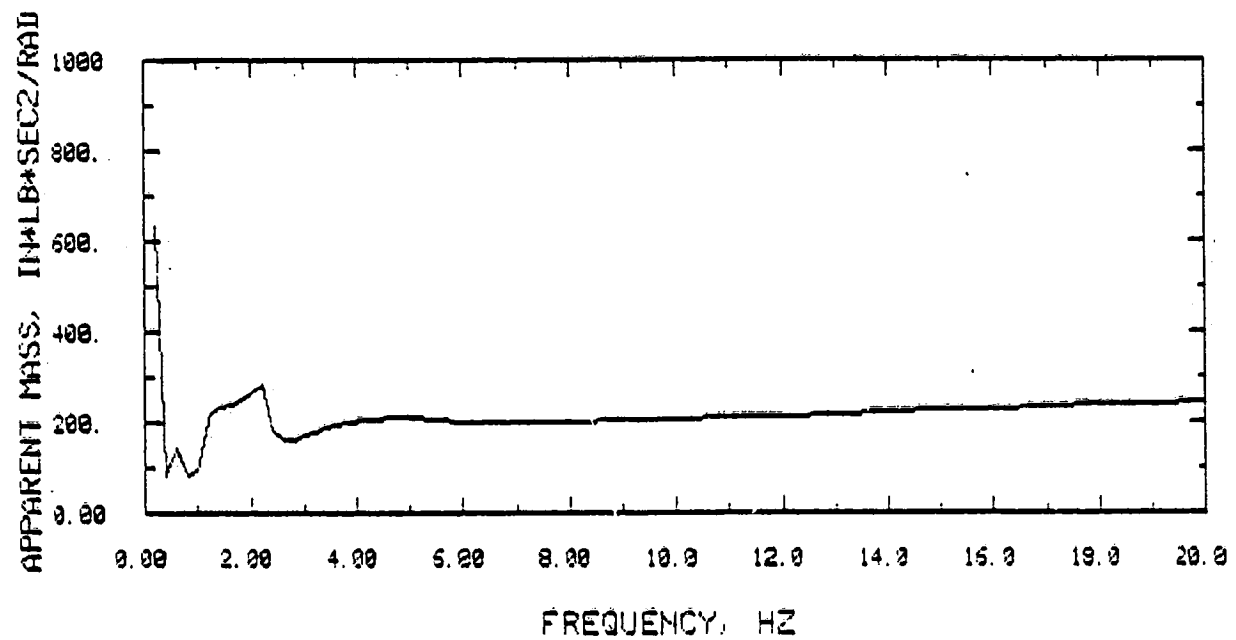
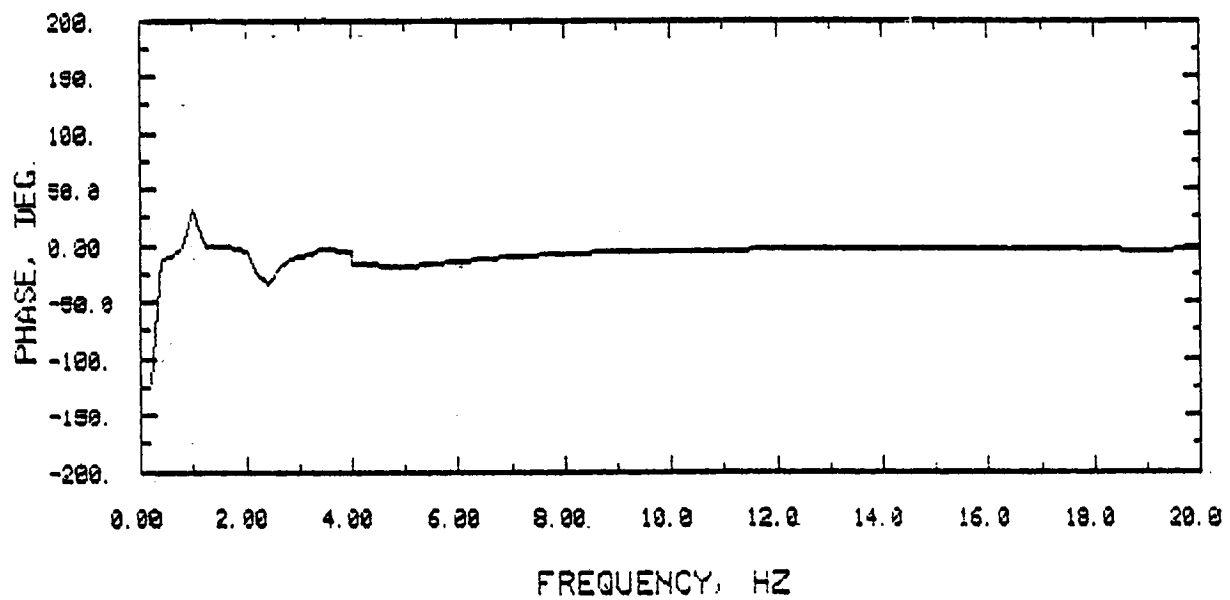
| | |
|----------------------|-------------------------|
| PLOT FIGURE | A2 |
| CONFIGURATION | LIQUID UNDER AIR ULLAGE |
| FILL / LIQUID | 100 GAL. / WATER |
| EXCITATION AMPLITUDE | 0.25 INCH P-P |
| EXCITATION AXIS-TYPE | X OR Y - TRANSLATION |



| | |
|----------------------|-------------------------|
| PLOT FIGURE | A3 |
| CONFIGURATION | LIQUID UNDER AIR ULLAGE |
| FILL / LIQUID | 108 GAL. / WATER |
| EXCITATION AMPLITUDE | 0.25 INCH P-P |
| EXCITATION AXIS-TYPE | X OR Y - ROTATION |

| FREQUENCY HZ | APPARENT MASS IN*LB*SEC ² /RAD | PHASE DEGREES |
|-----------------|--|------------------|
| 0.200 | 633.000 | -122.000 |
| 0.400 | 79.000 | -12.000 |
| 0.600 | 141.000 | -9.000 |
| 0.800 | 79.000 | -2.000 |
| 1.000 | 89.000 | 32.000 |
| 1.200 | 220.000 | 0.000 |
| 1.400 | 233.000 | -1.000 |
| 1.600 | 237.000 | -1.000 |
| 1.800 | 246.000 | -3.000 |
| 2.000 | 246.000 | -6.000 |
| 2.200 | 280.000 | -26.000 |
| 2.400 | 180.000 | -34.000 |
| 2.600 | 159.000 | -22.000 |
| 2.800 | 161.000 | -12.000 |
| 3.000 | 169.000 | -9.000 |
| 3.200 | 177.000 | -7.000 |
| 3.400 | 184.000 | -4.000 |
| 3.600 | 191.000 | -4.000 |
| 3.800 | 197.000 | -5.000 |
| 4.000 | 200.000 | -6.000 |
| 4.010 | 205.000 | -16.000 |
| 5.000 | 206.000 | -18.000 |
| 6.000 | 195.000 | -14.000 |
| 7.000 | 195.000 | -11.000 |
| 8.000 | 198.000 | -8.000 |
| 9.000 | 202.000 | -6.000 |
| 10.000 | 202.000 | -5.000 |
| 11.000 | 206.000 | -5.000 |
| 12.000 | 210.000 | -4.000 |
| 13.000 | 215.000 | -4.000 |
| 14.000 | 219.000 | -4.000 |
| 15.000 | 223.000 | -3.000 |
| 16.000 | 225.000 | -4.000 |
| 17.000 | 228.000 | -4.000 |
| 18.000 | 235.000 | -4.000 |
| 19.000 | 237.000 | -5.000 |
| 20.000 | 241.000 | -4.000 |

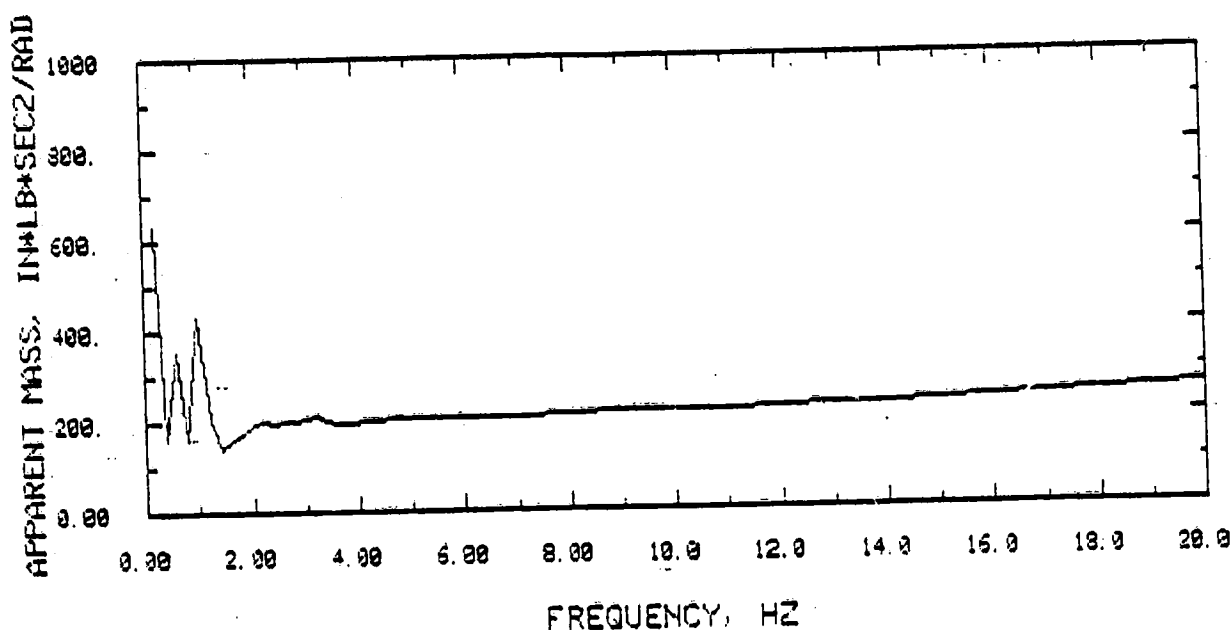
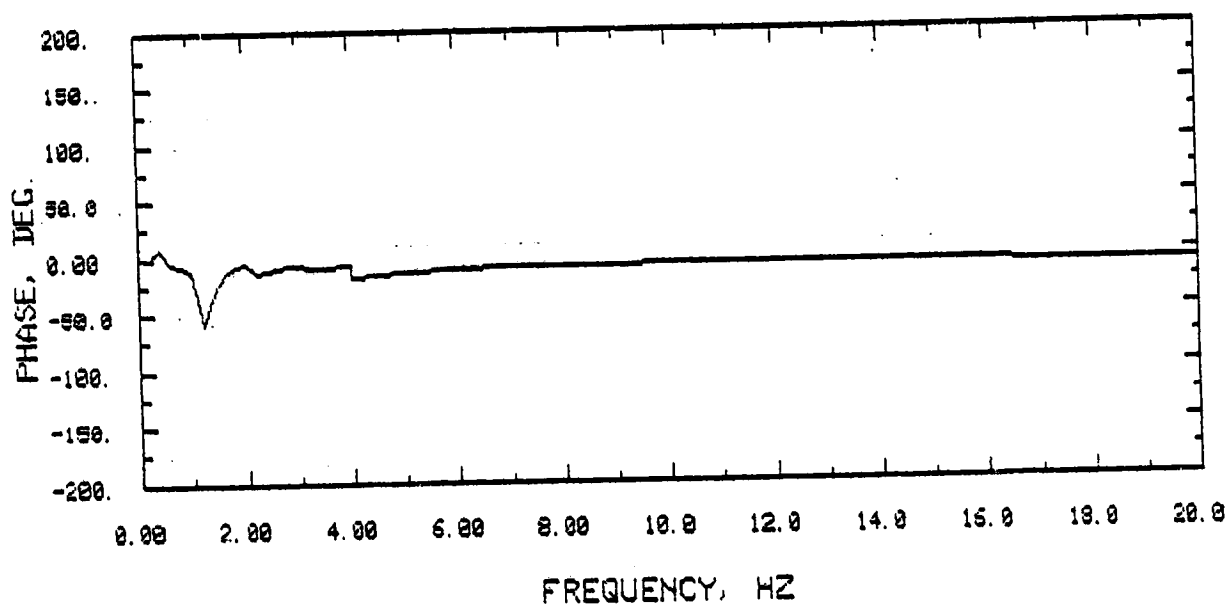
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|----------------------|-------------------------|
| PLOT FIGURE | A3 |
| CONFIGURATION | LIQUID UNDER AIR ULLAGE |
| FILL / LIQUID | 100 GAL. / WATER |
| EXCITATION AMPLITUDE | 0.25 INCH P-P |
| EXCITATION AXIS-TYPE | X OR Y - ROTATION |



PLOT FIGURE : A4
 CONFIGURATION : LIQUID UNDER WATER ULLAGE
 FILL / LIQUID : 108 GAL. / WATER
 EXCITATION AMPLITUDE : 0.25 INCH P-P
 EXCITATION AXIS-TYPE : X OR Y - ROTATION

| FREQUENCY | APPARENT MASS | PHASE |
|-----------|-----------------------------|---------|
| HZ | IN*LB*SEC ² /RAD | DEGREES |
| 0.200 | 433.000 | -2.000 |
| 0.400 | 158.000 | 7.000 |
| 0.600 | 352.000 | -5.000 |
| 0.800 | 158.000 | -8.000 |
| 1.000 | 431.000 | -12.000 |
| 1.200 | 202.000 | -58.000 |
| 1.400 | 136.000 | -30.000 |
| 1.600 | 158.000 | -14.000 |
| 1.800 | 172.000 | -8.000 |
| 2.000 | 190.000 | -6.000 |
| 2.200 | 196.000 | -15.000 |
| 2.400 | 189.000 | -13.000 |
| 2.600 | 197.000 | -10.000 |
| 2.800 | 200.000 | -8.000 |
| 3.000 | 205.000 | -8.000 |
| 3.200 | 208.000 | -10.000 |
| 3.400 | 199.000 | -11.000 |
| 3.600 | 193.000 | -9.000 |
| 3.800 | 191.000 | -8.000 |
| 4.000 | 192.000 | -7.000 |
| 4.010 | 196.000 | -18.000 |
| 5.000 | 202.000 | -14.000 |
| 6.000 | 202.000 | -12.000 |
| 7.000 | 202.000 | -11.000 |
| 8.000 | 207.000 | -9.000 |
| 9.000 | 214.000 | -9.000 |
| 10.000 | 212.000 | -8.000 |
| 11.000 | 215.000 | -7.000 |
| 12.000 | 219.000 | -7.000 |
| 13.000 | 224.000 | -7.000 |
| 14.000 | 227.000 | -8.000 |
| 15.000 | 232.000 | -8.000 |
| 16.000 | 238.000 | -8.000 |
| 17.000 | 244.000 | -9.000 |
| 18.000 | 248.000 | -10.000 |
| 19.000 | 252.000 | -11.000 |
| 20.000 | 261.000 | -11.000 |

PLOT FIGURE : A4
CONFIGURATION : LIQUID UNDER WATER ULLAGE
FILL / LIQUID : 100 GAL. / WATER
EXCITATION AMPLITUDE : 0.25 INCH P-P
EXCITATION AXIS-TYPE : X OR Y - ROTATION



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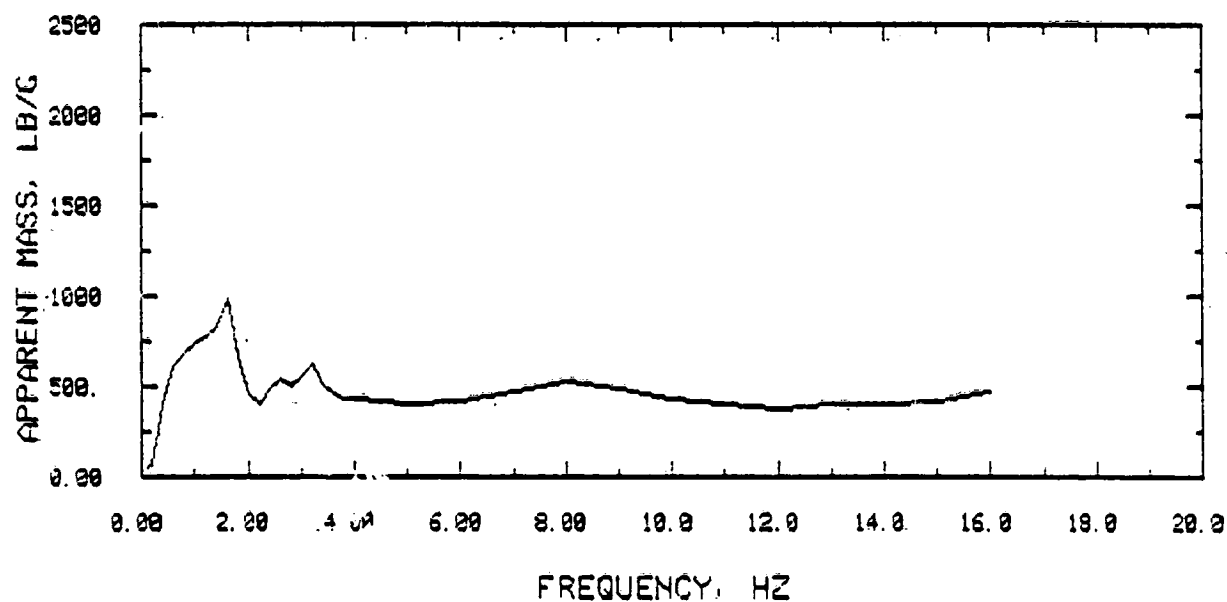
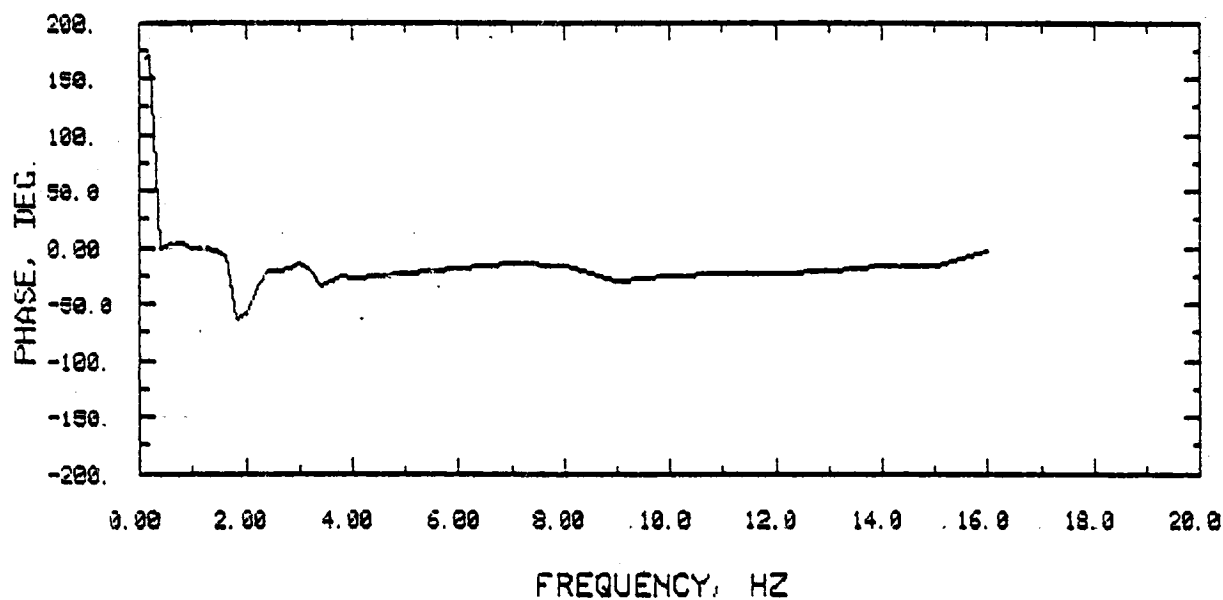
APPENDIX B

STEADY STATE HARMONIC RESPONSE DATA FOR
LIQUID-OVER-ULLAGE (AFT TANK) CONFIGURATIONS

PLOT FIGURE : B1
 CONFIGURATION : LIQUID OVER AIR ULLAGE
 FILL / LIQUID : 54 GAL. \ WATER
 EXCITATION AMPLITUDE : 0.25 INCH P-P
 EXCITATION AXIS-TYPE : X - TRANSLATION

| FREQUENCY HZ | APPARENT MASS LB/G | PHASE DEGREES |
|-----------------|-----------------------|------------------|
| 0.100 | 55.100 | 168.400 |
| 0.200 | 58.900 | 170.500 |
| 0.400 | 394.000 | -0.500 |
| 0.600 | 600.000 | 3.500 |
| 0.800 | 679.000 | 3.000 |
| 1.000 | 724.000 | -0.500 |
| 1.200 | 775.000 | 0.000 |
| 1.400 | 820.000 | -3.500 |
| 1.600 | 978.000 | -7.500 |
| 1.800 | 653.000 | -64.000 |
| 2.000 | 450.000 | -58.000 |
| 2.200 | 393.000 | -35.000 |
| 2.400 | 479.000 | -21.000 |
| 2.600 | 534.000 | -22.000 |
| 2.800 | 500.000 | -19.000 |
| 3.000 | 545.000 | -14.500 |
| 3.200 | 615.000 | -21.000 |
| 3.400 | 503.000 | -33.500 |
| 3.600 | 445.000 | -29.000 |
| 3.800 | 423.000 | -25.000 |
| 4.000 | 420.000 | -28.000 |
| 5.000 | 395.000 | -23.000 |
| 6.000 | 415.000 | -18.000 |
| 7.000 | 460.000 | -15.000 |
| 8.000 | 520.000 | -17.500 |
| 9.000 | 475.000 | -31.000 |
| 10.000 | 425.000 | -26.000 |
| 11.000 | 395.000 | -23.500 |
| 12.000 | 375.000 | -23.000 |
| 13.000 | 395.000 | -21.000 |
| 14.000 | 390.000 | -16.000 |
| 15.000 | 415.000 | -16.500 |
| 16.000 | 470.000 | -3.000 |

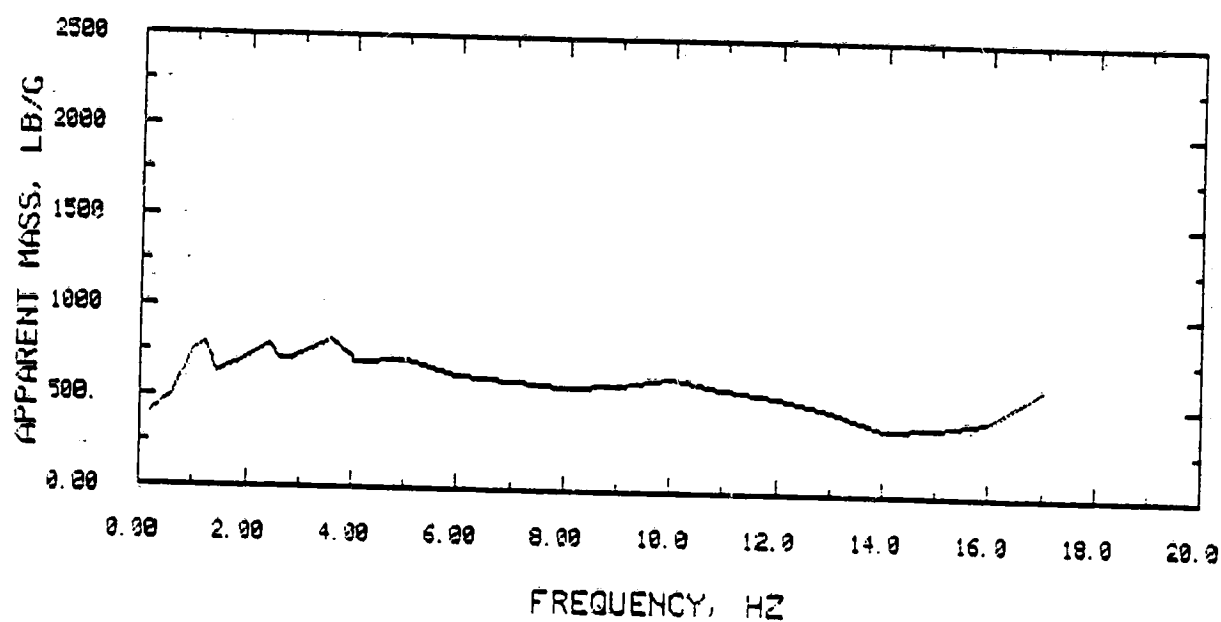
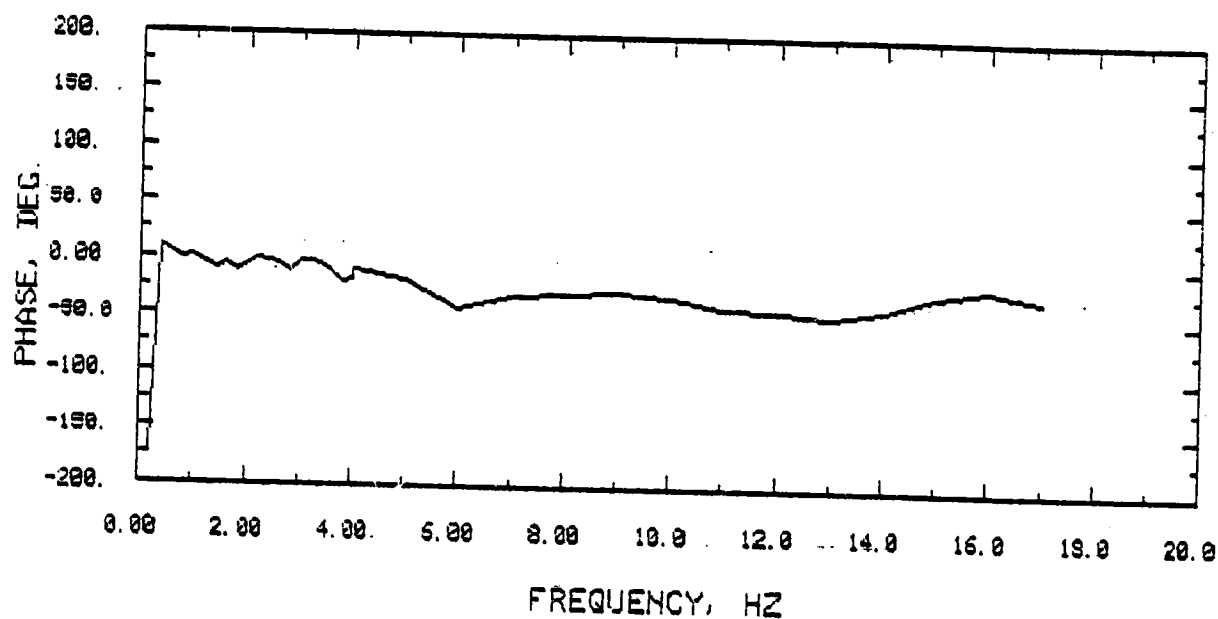
PLOT FIGURE : 81
CONFIGURATION : LIQUID OVER AIR ULLAGE
FILL / LIQUID : 54 GAL. \ WATER
EXCITATION AMPLITUDE : 0.25 INCH P-P
EXCITATION AXIS-TYPE : X - TRANSLATION



PLOT FIGURE : B2
 CONFIGURATION : LIQUID OVER AIR ULLAGE
 FILL / LIQUID : 54 GAL. / WATER
 EXCITATION AMPLITUDE : 0.25 INCH P-P
 EXCITATION AXIS-TYPE : Y - TRANSLATION

| FREQUENCY HZ | APPARENT MASS LB/G | PHASE DEGREES |
|-----------------|-----------------------|------------------|
| 0.200 | 400.000 | -175.000 |
| 0.400 | 450.000 | 9.000 |
| 0.600 | 500.000 | 4.000 |
| 0.800 | 620.000 | 0.000 |
| 1.000 | 750.000 | 0.500 |
| 1.200 | 788.000 | -6.000 |
| 1.400 | 612.000 | -9.500 |
| 1.600 | 656.000 | -5.000 |
| 1.800 | 675.000 | -12.000 |
| 2.200 | 737.000 | 0.000 |
| 2.400 | 780.000 | -4.000 |
| 2.600 | 697.000 | -6.500 |
| 2.800 | 700.000 | -13.000 |
| 3.000 | 728.000 | -3.000 |
| 3.200 | 753.000 | -4.000 |
| 3.400 | 782.000 | -6.000 |
| 3.600 | 812.000 | -13.000 |
| 3.800 | 753.000 | -20.000 |
| 4.000 | 710.000 | -19.000 |
| 4.010 | 690.000 | -11.000 |
| 5.000 | 705.000 | -19.000 |
| 6.000 | 615.000 | -42.500 |
| 7.000 | 590.000 | -33.000 |
| 8.000 | 560.000 | -29.000 |
| 9.000 | 575.000 | -27.000 |
| 10.000 | 620.000 | -32.000 |
| 11.000 | 565.000 | -40.000 |
| 12.000 | 515.000 | -44.000 |
| 13.000 | 450.000 | -47.000 |
| 14.000 | 350.000 | -41.000 |
| 15.000 | 370.000 | -27.000 |
| 16.000 | 405.000 | -21.000 |
| 17.000 | 590.000 | -29.000 |

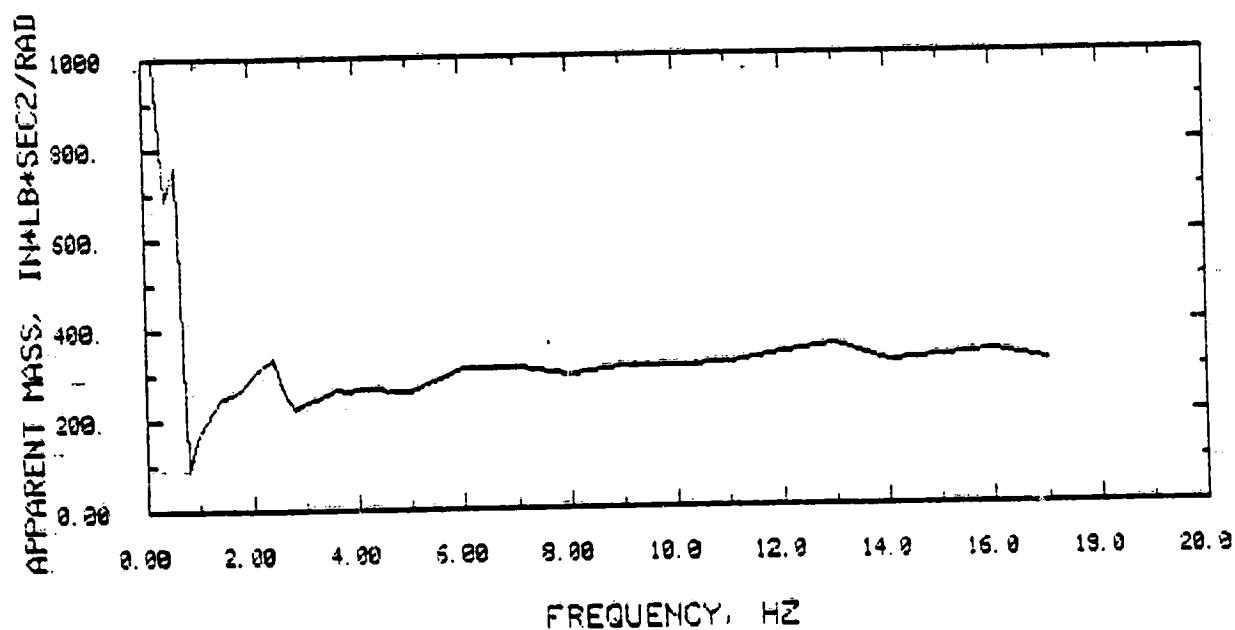
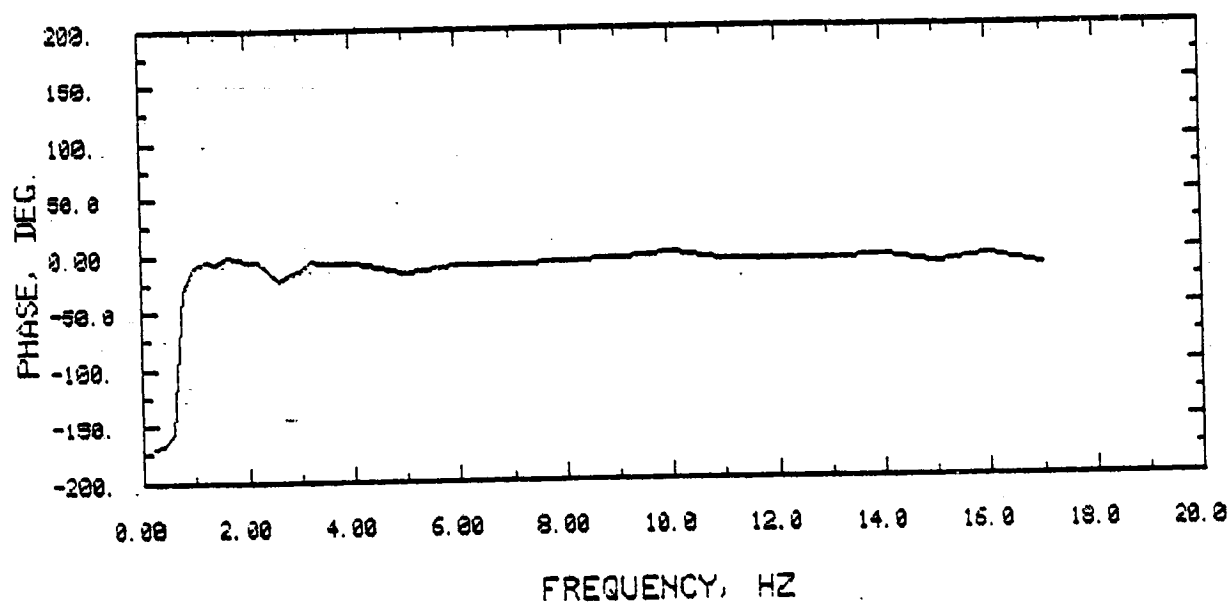
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|----------------------|---|------------------------|
| PLOT FIGURE | : | 82 |
| CONFIGURATION | : | LIQUID OVER AIR ULLAGE |
| FILL / LIQUID | : | 54 GAL. / WATER |
| EXCITATION AMPLITUDE | : | 0.25 INCH P-P |
| EXCITATION AXIS-TYPE | : | Y - TRANSLATION |



PLOT FIGURE : 33
 CONFIGURATION : LIQUID OVER AIR ULLAGE
 FILL / LIQUID : 54 GAL. WATER
 EXCITATION AMPLITUDE : 0.25 INCH P-P
 EXCITATION AXIS-TYPE : Z - ROTATION

| FREQUENCY | APPARENT MASS | PHASE |
|-----------|-----------------------------|----------|
| HZ | IN*LB*SEC ² /RAD | DEGREES |
| 0.200 | 4104.000 | -169.000 |
| 0.400 | 684.000 | -167.000 |
| 0.600 | 756.000 | -156.000 |
| 0.800 | 86.000 | -30.000 |
| 1.000 | 171.000 | -10.000 |
| 1.200 | 213.000 | -6.000 |
| 1.400 | 246.000 | -7.000 |
| 1.600 | 254.000 | -2.000 |
| 1.800 | 271.000 | -3.000 |
| 2.000 | 297.000 | -5.000 |
| 2.200 | 319.000 | -6.000 |
| 2.400 | 330.000 | -14.000 |
| 2.600 | 256.000 | -23.000 |
| 2.800 | 222.000 | -16.000 |
| 3.000 | 234.000 | -14.000 |
| 3.200 | 242.000 | -6.000 |
| 3.400 | 254.000 | -7.000 |
| 3.600 | 263.000 | -7.000 |
| 3.800 | 259.000 | -8.000 |
| 4.000 | 265.000 | -7.000 |
| 5.000 | 257.000 | -16.000 |
| 6.000 | 308.000 | -10.000 |
| 7.000 | 308.000 | -10.000 |
| 8.000 | 291.000 | -8.500 |
| 9.000 | 308.000 | -6.000 |
| 10.000 | 308.000 | -1.000 |
| 11.000 | 316.000 | -8.000 |
| 12.000 | 334.000 | -8.000 |
| 13.000 | 351.000 | -8.000 |
| 14.000 | 316.000 | -6.000 |
| 15.000 | 325.000 | -12.000 |
| 16.000 | 334.000 | -5.000 |
| 17.000 | 316.000 | -14.000 |

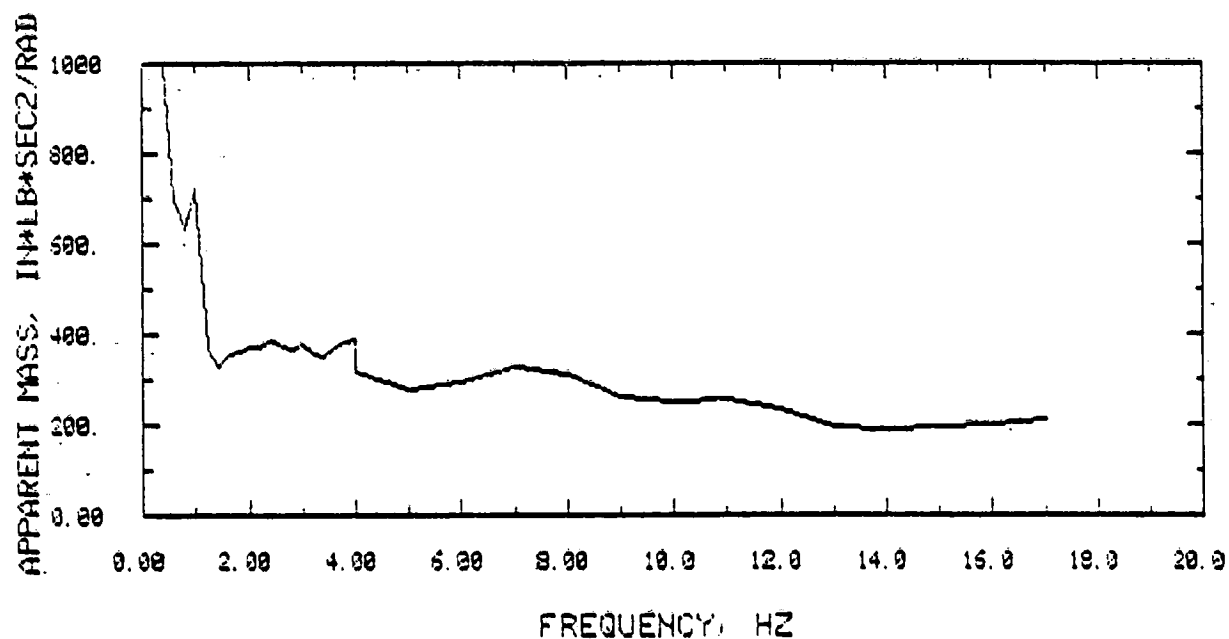
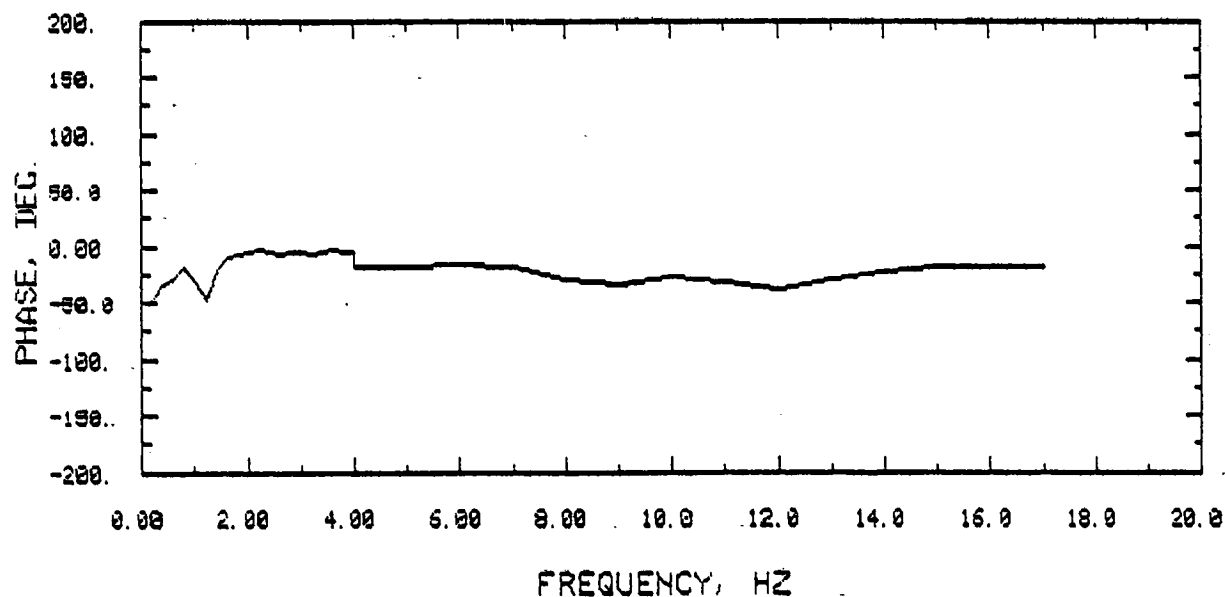
PLOT FIGURE : 83
CONFIGURATION : LIQUID OVER AIR ULLAGE
FILL / LIQUID : 54 GAL. WATER
EXCITATION AMPLITUDE : 0.25 INCH P-P
EXCITATION AXIS-TYPE : 2 - ROTATION



| | | |
|----------------------|---|------------------------|
| PLOT FIGURE | : | B4 |
| CONFIGURATION | : | LIQUID OVER AIR ULLAGE |
| FILL / LIQUID | : | 54 GAL. / WATER |
| EXCITATION AMPLITUDE | : | 0.25 INCH P-P |
| EXCITATION AXIS-TYPE | : | X - ROTATION |

| FREQUENCY | APPARENT MASS | PHASE |
|-----------|-----------------------------|---------|
| HZ | IN*LB*SEC ² /RAD | DEGREES |
| 0.200 | 2994.000 | -48.000 |
| 0.400 | 1266.000 | -35.000 |
| 0.600 | 695.000 | -30.000 |
| 0.800 | 631.000 | -18.000 |
| 1.000 | 718.000 | -29.000 |
| 1.200 | 362.000 | -48.000 |
| 1.400 | 327.000 | -21.000 |
| 1.600 | 354.000 | -11.000 |
| 1.800 | 358.000 | -7.000 |
| 2.000 | 367.000 | -5.000 |
| 2.200 | 370.000 | -4.000 |
| 2.400 | 385.000 | -6.000 |
| 2.600 | 372.000 | -7.000 |
| 2.800 | 362.000 | -6.000 |
| 3.000 | 373.000 | -6.000 |
| 3.200 | 353.000 | -7.000 |
| 3.400 | 348.000 | -5.000 |
| 3.600 | 372.000 | -4.000 |
| 3.800 | 380.000 | -5.000 |
| 4.000 | 384.000 | -5.000 |
| 4.010 | 316.000 | -18.000 |
| 5.000 | 277.000 | -18.000 |
| 6.000 | 292.000 | -16.000 |
| 7.000 | 323.000 | -17.000 |
| 8.000 | 308.000 | -29.000 |
| 9.000 | 261.000 | -34.000 |
| 10.000 | 246.000 | -28.000 |
| 11.000 | 254.000 | -33.000 |
| 12.000 | 231.000 | -38.000 |
| 13.000 | 192.000 | -31.000 |
| 14.000 | 184.000 | -23.000 |
| 15.000 | 192.000 | -19.000 |
| 16.000 | 200.000 | -18.000 |
| 17.000 | 207.000 | -18.000 |

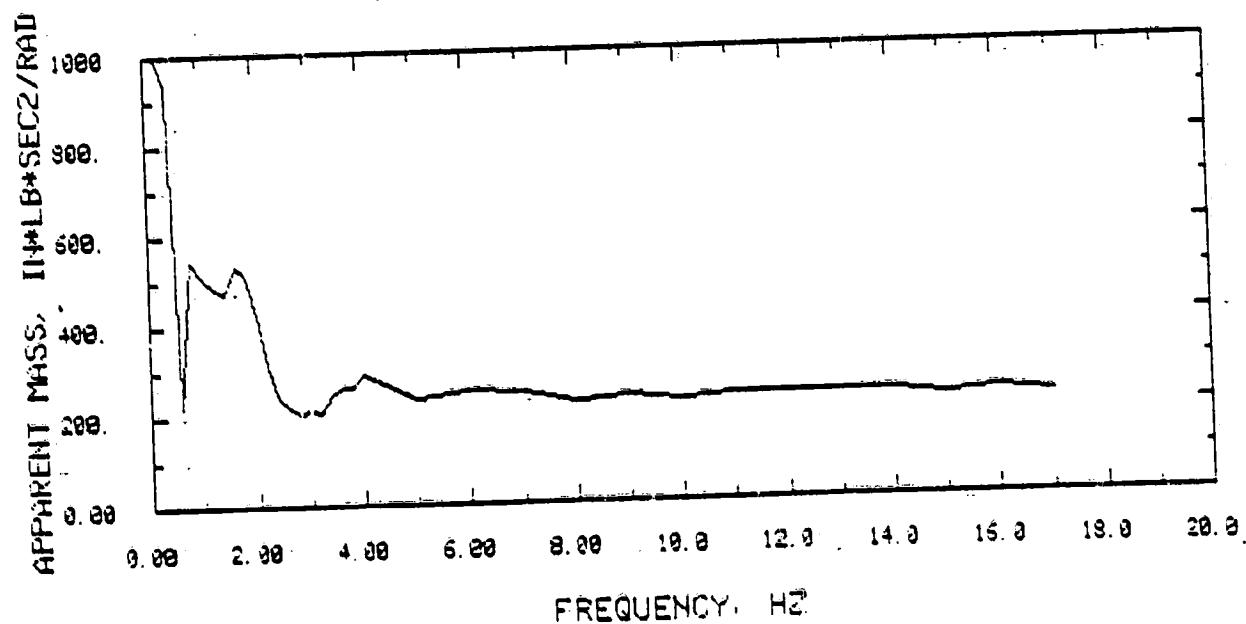
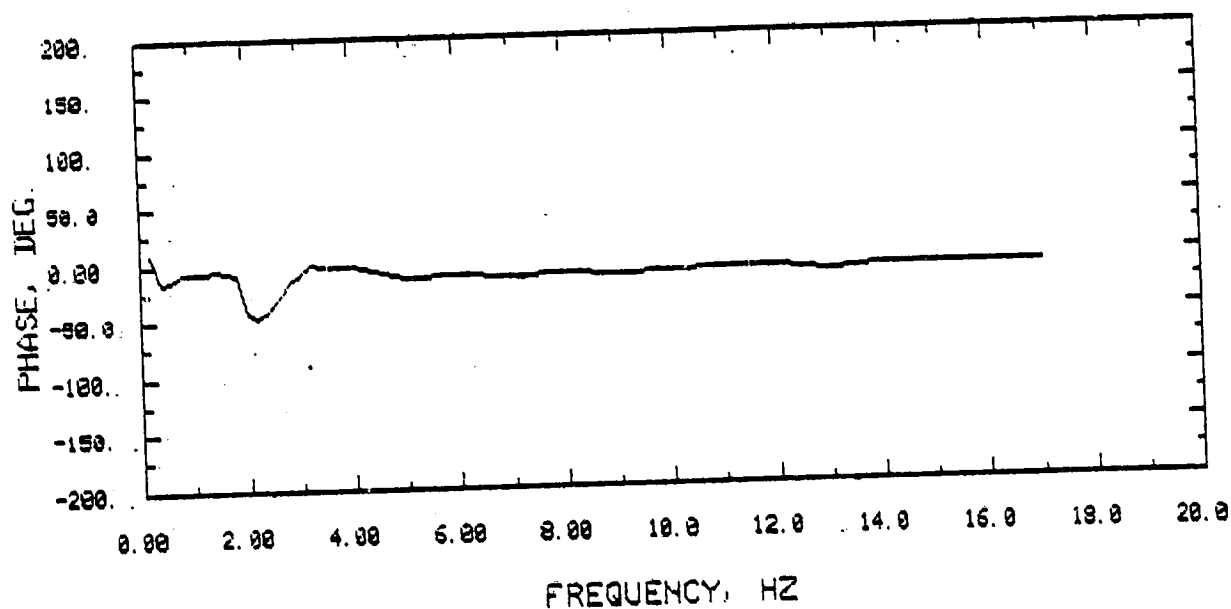
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| PLOT FIGURE | : | 84 |
| CONFIGURATION | : | LIQUID OVER AIR ULLAGE |
| FILL / LIQUID | : | 54 GAL. / WATER |
| EXCITATION AMPLITUDE | : | 0.25 INCH P-P |
| EXCITATION AXIS-TYPE | : | X - ROTATION |



| | |
|----------------------|------------------------|
| PLOT FIGURE | B5 |
| CONFIGURATION | LIQUID OVER AIR ULLAGE |
| FILL / LIQUID | 54 GAL. / WATER |
| EXCITATION AMPLITUDE | 0.25 INCH P-P |
| EXCITATION AXIS-TYPE | Y - ROTATION |

| FREQUENCY | APPARENT MASS | PHASE |
|-----------|-----------------------------|---------|
| HZ | IN*LB*SEC ² /RAD | DEGREES |
| 0.200 | 3906.000 | 9.000 |
| 0.400 | 933.000 | -17.000 |
| 0.600 | 217.000 | -15.000 |
| 0.800 | 540.700 | -7.000 |
| 1.000 | 510.000 | -7.000 |
| 1.200 | 481.000 | -7.000 |
| 1.400 | 470.000 | -5.000 |
| 1.600 | 529.000 | -7.000 |
| 1.800 | 511.000 | -9.000 |
| 2.000 | 417.000 | -44.000 |
| 2.200 | 308.000 | -47.000 |
| 2.400 | 238.000 | -40.000 |
| 2.600 | 212.000 | -29.000 |
| 2.800 | 205.000 | -17.000 |
| 3.000 | 208.000 | -9.000 |
| 3.200 | 204.000 | -2.000 |
| 3.400 | 243.000 | -4.000 |
| 3.600 | 259.000 | -3.000 |
| 3.800 | 259.000 | -3.000 |
| 4.000 | 284.000 | -4.000 |
| 5.000 | 231.000 | -14.000 |
| 6.000 | 248.000 | -12.000 |
| 7.000 | 239.000 | -15.000 |
| 8.000 | 222.000 | -13.000 |
| 9.000 | 231.000 | -14.000 |
| 10.000 | 222.000 | -12.000 |
| 11.000 | 231.000 | -10.000 |
| 12.000 | 231.000 | -11.000 |
| 13.000 | 231.000 | -14.000 |
| 14.000 | 231.000 | -11.000 |
| 15.000 | 222.000 | -10.000 |
| 16.000 | 231.000 | -10.000 |
| 17.000 | 222.000 | -10.000 |

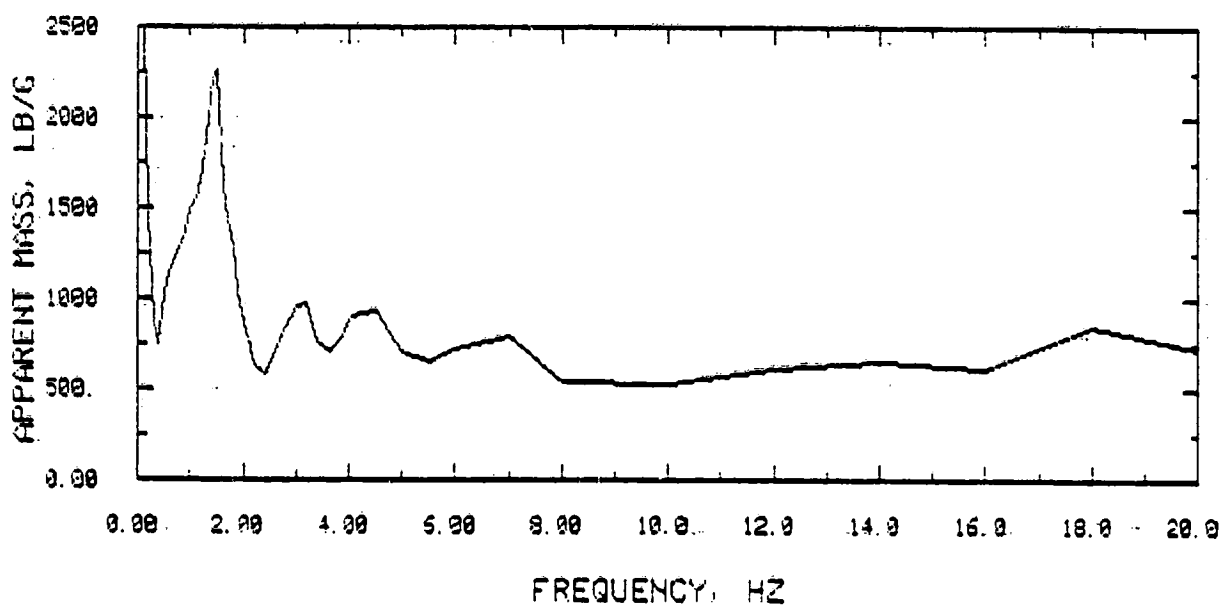
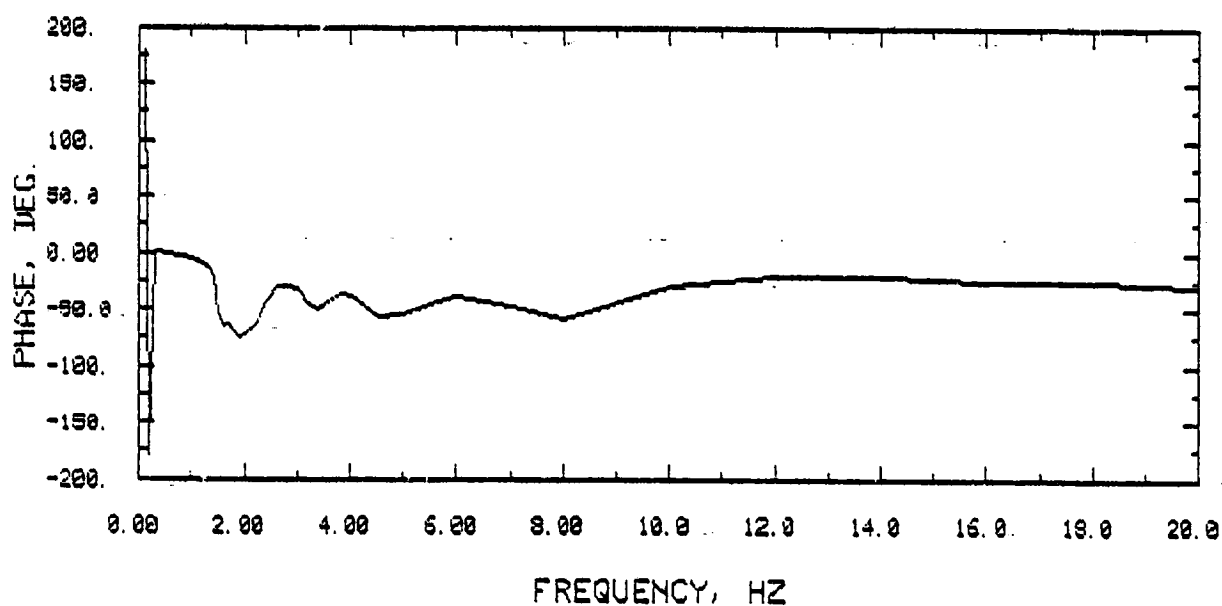
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|----------------------|------------------------|
| PLOT FIGURE | 35 |
| CONFIGURATION | LIQUID OVER AIR ULLAGE |
| FILL / LIQUID | 54 GAL. / WATER |
| EXCITATION AMPLITUDE | 0.25 INCH P-P |
| EXCITATION AXIS-TYPE | Y - ROTATION |



PLOT FIGURE : B6
 CONFIGURATION : LIQUID OVER AIR ULLAGE
 FILL / LIQUID : 54 GAL. 2.52 SG ZNBR2
 EXCITATION AMPLITUDE : 0.10 INCH P-P
 EXCITATION AXIS-TYPE : X - TRANSLATION

| FREQUENCY HZ | APPARENT MASS LB/G | PHASE DEGREES |
|-----------------|-----------------------|------------------|
| 0.100 | 9999.000 | 178.000 |
| 0.200 | 1463.000 | -178.000 |
| 0.299 | 865.000 | 2.000 |
| 0.399 | 737.000 | 2.000 |
| 0.500 | 1017.000 | -2.000 |
| 0.600 | 1141.000 | -2.000 |
| 0.701 | 1234.000 | -3.000 |
| 0.802 | 1307.000 | -3.000 |
| 0.903 | 1368.000 | -4.000 |
| 1.000 | 1492.000 | -5.000 |
| 1.100 | 1552.000 | -7.000 |
| 1.201 | 1722.000 | -10.000 |
| 1.302 | 1892.000 | -12.000 |
| 1.404 | 2222.000 | -22.000 |
| 1.505 | 2257.000 | -55.000 |
| 1.606 | 1622.000 | -65.000 |
| 1.708 | 1421.000 | -63.000 |
| 1.810 | 1283.000 | -71.000 |
| 1.910 | 1029.000 | -75.000 |
| 2.010 | 860.000 | -73.000 |
| 2.210 | 635.000 | -64.000 |
| 2.410 | 579.000 | -43.000 |
| 2.610 | 715.000 | -30.000 |
| 2.810 | 847.000 | -29.000 |
| 3.010 | 945.000 | -33.000 |
| 3.210 | 960.000 | -46.000 |
| 3.410 | 753.000 | -49.000 |
| 3.620 | 704.000 | -42.000 |
| 3.820 | 770.000 | -37.000 |
| 4.020 | 897.000 | -38.000 |
| 4.500 | 920.000 | -36.000 |
| 5.000 | 695.000 | -54.000 |
| 5.500 | 640.000 | -45.000 |
| 6.000 | 710.000 | -38.000 |
| 7.000 | 780.000 | -48.000 |
| 8.000 | 535.000 | -59.000 |
| 10.000 | 525.000 | -30.000 |
| 12.000 | 610.000 | -20.000 |
| 14.000 | 645.000 | -21.000 |
| 16.000 | 600.000 | -25.000 |
| 18.000 | 840.000 | -26.000 |
| 20.000 | 710.000 | -30.000 |

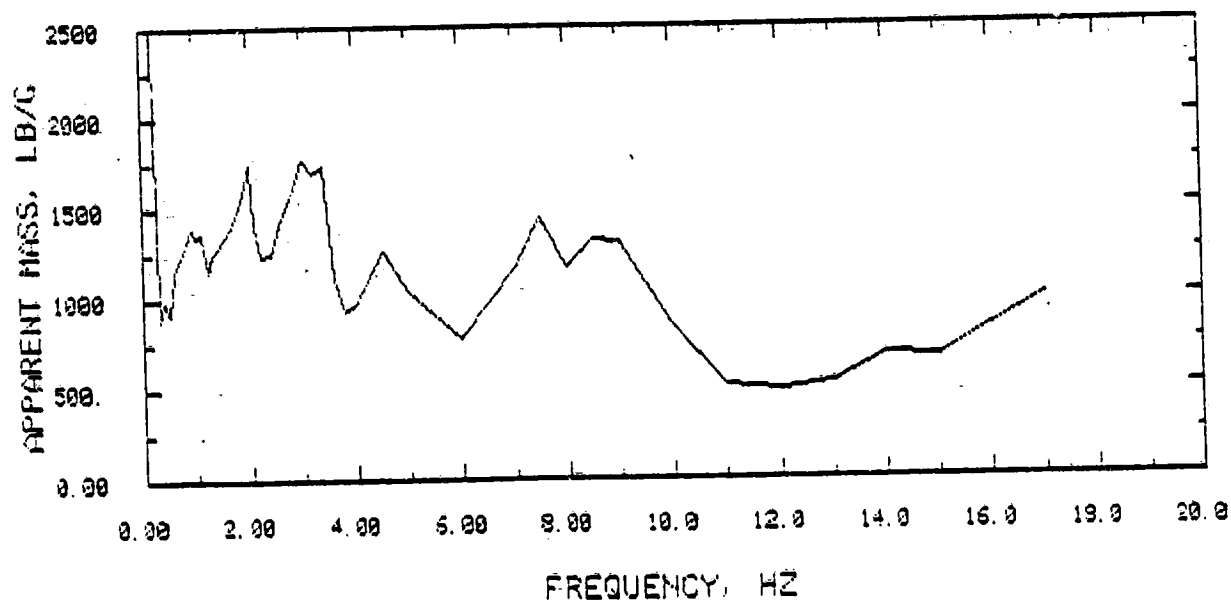
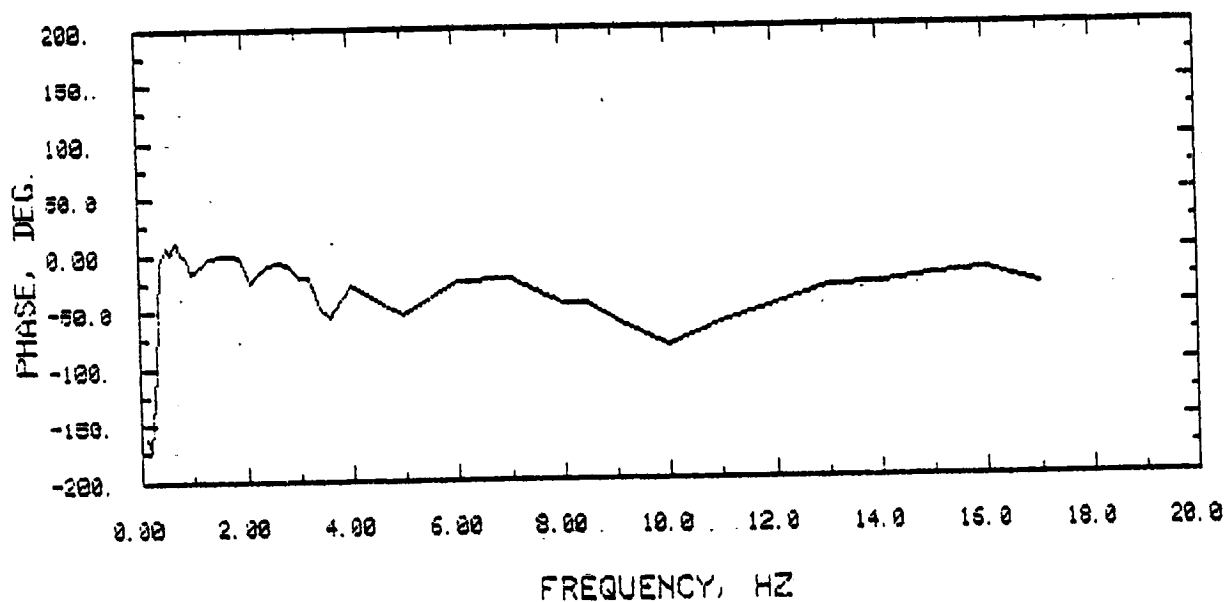
PLOT FIGURE : 56
CONFIGURATION : LIQUID OVER AIR ULLAGE
FILL / LIQUID : 54 GAL. 2.52 SG 2NBR2
EXCITATION AMPLITUDE : 0.10 INCH P-P
EXCITATION AXIS-TYPE : X - TRANSLATION



PLOT FIGURE : 87
 CONFIGURATION : LIQUID OVER AIR ULLAGE
 FILL / LIQUID : 54 GAL. / 2.52 SQ ZNBR2
 EXCITATION AMPLITUDE : 0.10 INCH P-F
 EXCITATION AXIS-TYPE : Y - TRANSLATION

| FREQUENCY HZ | APPARENT MASS LB/O | PHASE DEGREES |
|-----------------|-----------------------|------------------|
| 0.100 | 14000.000 | -161.000 |
| 0.200 | 3000.000 | -173.000 |
| 0.300 | 888.000 | -102.000 |
| 0.400 | 974.000 | -8.000 |
| 0.500 | 916.000 | 7.500 |
| 0.600 | 1146.000 | 1.500 |
| 0.700 | 1250.000 | 12.500 |
| 0.800 | 1280.000 | 1.000 |
| 0.900 | 1396.000 | -4.000 |
| 1.000 | 1340.000 | -16.500 |
| 1.100 | 1360.000 | -12.500 |
| 1.200 | 1150.000 | -7.500 |
| 1.300 | 1246.000 | -4.000 |
| 1.400 | 1280.000 | -2.500 |
| 1.500 | 1330.000 | -2.000 |
| 1.600 | 1388.000 | -1.500 |
| 1.700 | 1442.000 | -2.000 |
| 1.800 | 1508.000 | -2.000 |
| 1.900 | 1594.000 | -4.000 |
| 2.000 | 1746.000 | -15.500 |
| 2.100 | 1396.000 | -25.000 |
| 2.200 | 1224.000 | -19.000 |
| 2.400 | 1248.000 | -9.000 |
| 2.600 | 1448.000 | -8.500 |
| 2.800 | 1574.000 | -10.000 |
| 3.000 | 1776.000 | -22.000 |
| 3.200 | 1692.000 | -21.000 |
| 3.400 | 1728.000 | -47.500 |
| 3.600 | 1098.000 | -56.000 |
| 3.800 | 922.000 | -40.000 |
| 4.000 | 964.000 | -27.500 |
| 4.500 | 1262.500 | -44.000 |
| 5.000 | 1037.500 | -55.000 |
| 6.000 | 775.000 | -24.500 |
| 7.000 | 1164.400 | -22.500 |
| 7.500 | 1437.500 | -33.500 |
| 8.000 | 1160.400 | -46.500 |
| 8.500 | 1308.000 | -44.500 |
| 9.000 | 1300.000 | -61.500 |
| 10.000 | 842.600 | -83.000 |
| 11.000 | 503.000 | -63.500 |
| 12.000 | 475.000 | -47.000 |
| 13.000 | 518.600 | -32.000 |
| 14.000 | 675.000 | -29.500 |
| 15.000 | 662.400 | -22.500 |
| 16.000 | 825.000 | -19.500 |
| 17.000 | 993.600 | -31.500 |

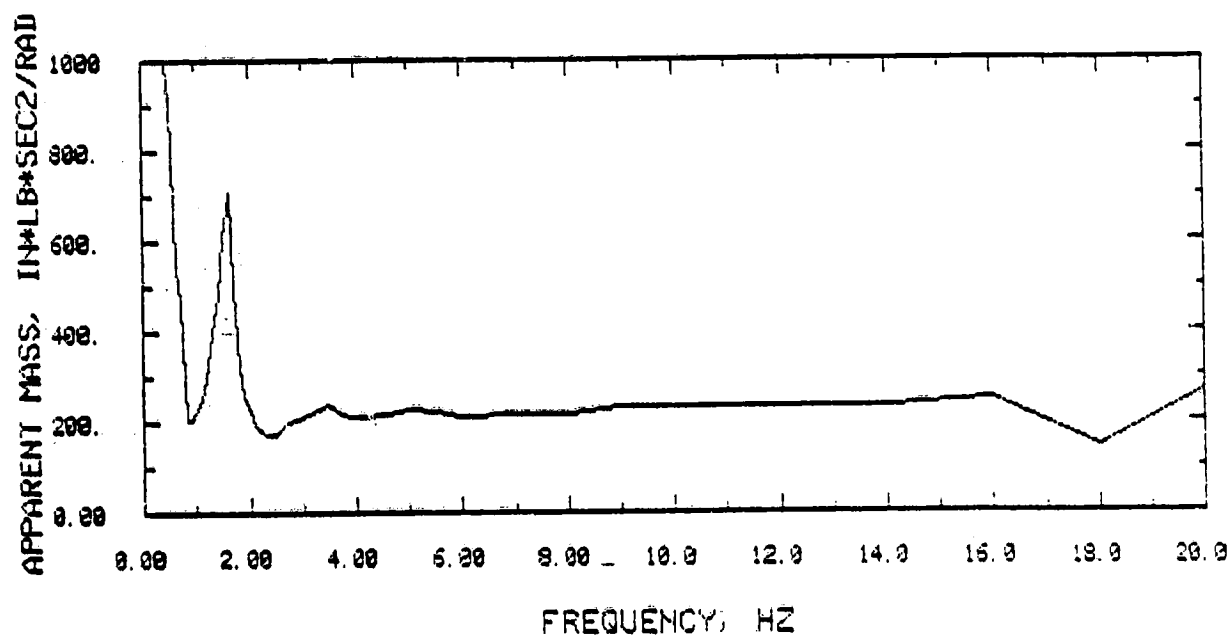
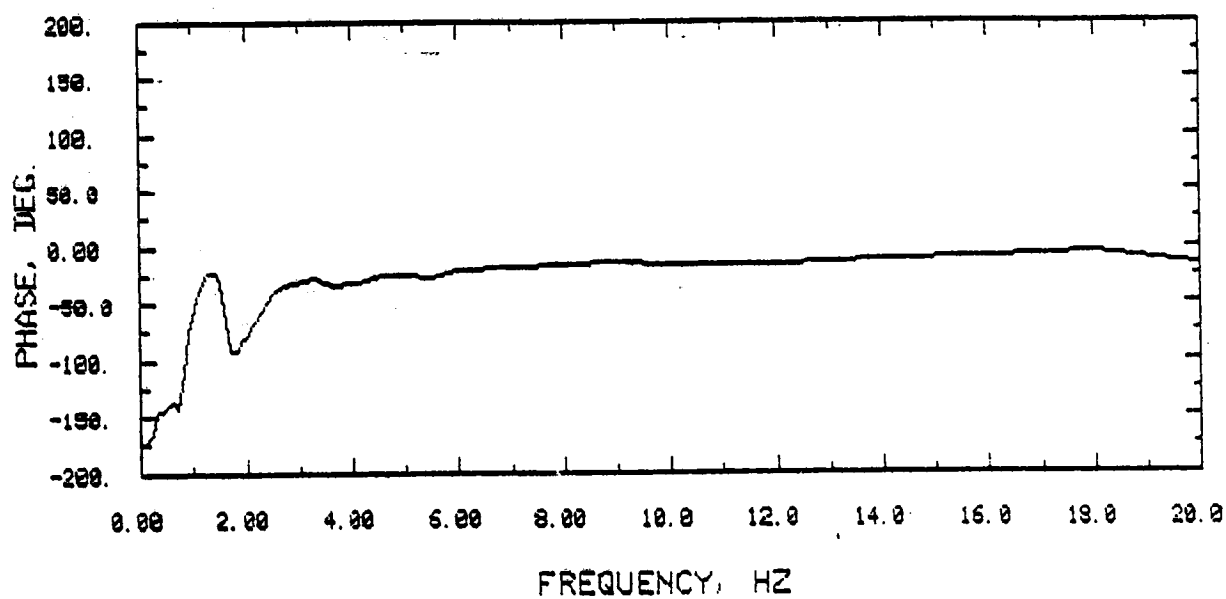
PLOT FIGURE 87
CONFIGURATION : LIQUID OVER-AIR ULLAGE
FILL / LIQUID : 54 GAL. / 2.52 SG ZNBR2
EXCITATION AMPLITUDE : 0.10 INCH P-P
EXCITATION AXIS-TYPE : Y - TRANSLATION



PLOT FIGURE : 88
 CONFIGURATION : LIQUID OVER AIR ULLAGE
 FILL / LIQUID : 54 GAL. / 2.52 SQ ZNBR2
 EXCITATION AMPLITUDE : 0.10 INCH P-P
 EXCITATION AXIS TYPE : Z - ROTATION

| FREQUENCY HZ | APPARENT MASS IN*LB*SEC ² /RAD | PHASE DEGREES |
|-----------------|--|------------------|
| 0.110 | 22122.852 | -172.000 |
| 0.210 | 7587.486 | -169.000 |
| 0.310 | 3481.875 | -146.000 |
| 0.410 | 1791.477 | -146.000 |
| 0.510 | 900.522 | -141.000 |
| 0.610 | 539.546 | -137.000 |
| 0.710 | 464.641 | -144.000 |
| 0.810 | 203.998 | -109.000 |
| 0.910 | 202.034 | -75.000 |
| 1.100 | 248.882 | -39.000 |
| 1.200 | 325.313 | -25.000 |
| 1.300 | 395.986 | -23.000 |
| 1.400 | 478.012 | -24.000 |
| 1.500 | 609.730 | -33.000 |
| 1.600 | 705.814 | -66.000 |
| 1.700 | 497.860 | -93.000 |
| 1.800 | 320.150 | -93.000 |
| 1.900 | 250.261 | -83.000 |
| 2.000 | 225.860 | -78.000 |
| 2.100 | 189.687 | -71.000 |
| 2.300 | 170.783 | -57.000 |
| 2.500 | 171.319 | -40.000 |
| 2.700 | 197.368 | -34.000 |
| 2.900 | 202.913 | -32.000 |
| 3.100 | 212.394 | -29.000 |
| 3.300 | 227.374 | -28.000 |
| 3.500 | 234.909 | -33.000 |
| 3.700 | 215.088 | -35.000 |
| 3.900 | 208.993 | -33.000 |
| 4.100 | 207.015 | -32.000 |
| 4.600 | 215.000 | -25.000 |
| 5.000 | 225.000 | -26.000 |
| 5.500 | 218.000 | -28.000 |
| 6.000 | 211.000 | -20.000 |
| 7.000 | 212.000 | -18.000 |
| 8.000 | 215.000 | -16.000 |
| 9.000 | 221.000 | -15.000 |
| 10.000 | 230.000 | -16.000 |
| 12.000 | 233.000 | -16.000 |
| 14.000 | 228.000 | -12.000 |
| 16.000 | 248.000 | -11.000 |
| 18.000 | 140.000 | -5.000 |
| 20.000 | 262.000 | -16.000 |

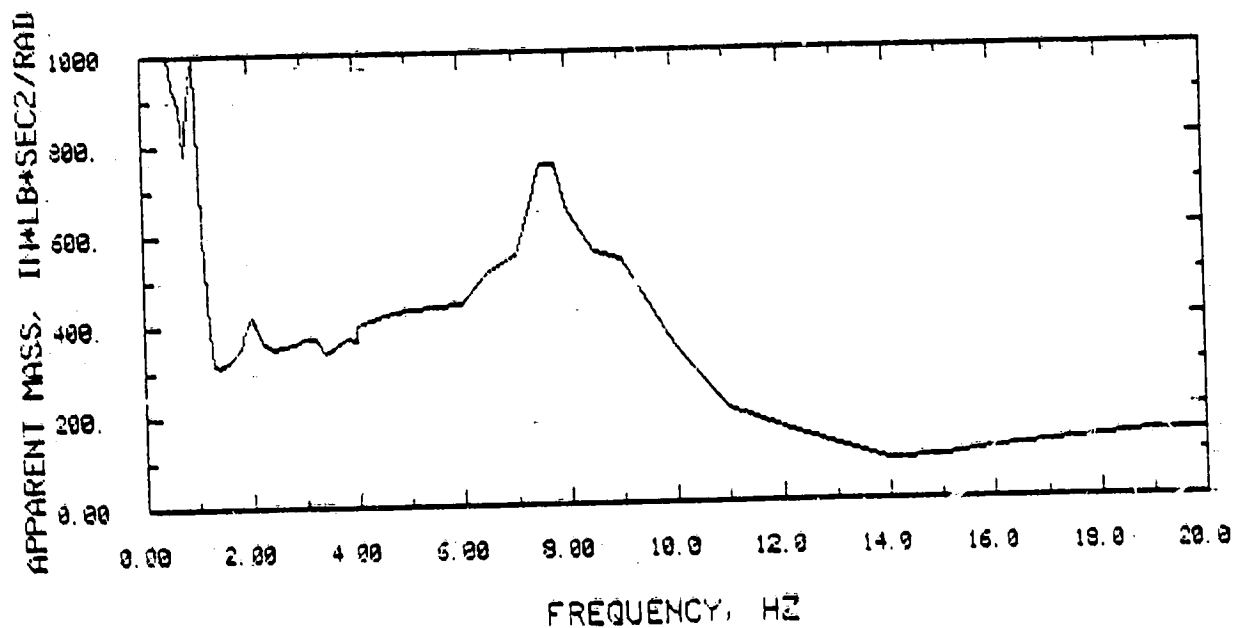
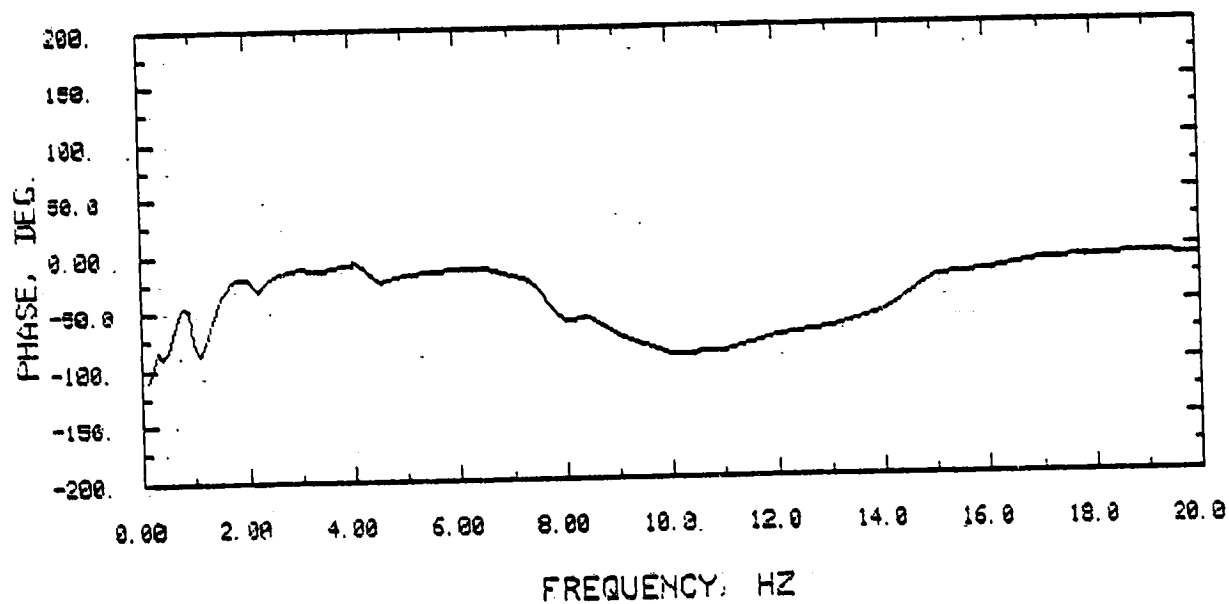
PLOT FIGURE : 88
CONFIGURATION : LIQUID OVER AIR ULLAGE
FILL / LIQUID : 54 GAL. / 2.52 SG ZHBR2
EXCITATION AMPLITUDE : 0.10 INCH P-P
EXCITATION AXIS TYPE : Z - ROTATION



PLOT FIGURE : 89
 CONFIGURATION : LIQUID OVER AIR ULLAGE
 FILL / LIQUID : 54 GAL. / 2.52 SG ZNBR2
 EXCITATION AMPLITUDE : 0.10 INCH P-P
 EXCITATION AXIS-TYPE : X - ROTATION

| FREQUENCY | APPARENT MASS | PHASE |
|-----------|-----------------------------|----------|
| HZ | IN*LB*SEC ² /RAD | DEGREES |
| 0.100 | 36718.000 | -111.000 |
| 0.200 | 7700.000 | -104.000 |
| 0.300 | 4456.000 | -83.000 |
| 0.400 | 1668.000 | -90.000 |
| 0.500 | 1203.000 | -84.000 |
| 0.600 | 929.000 | -69.000 |
| 0.700 | 887.000 | -52.000 |
| 0.800 | 783.000 | -45.000 |
| 0.900 | 950.000 | -48.000 |
| 1.000 | 1036.000 | -77.000 |
| 1.100 | 609.000 | -88.000 |
| 1.200 | 394.000 | -77.000 |
| 1.300 | 316.000 | -63.000 |
| 1.400 | 307.000 | -47.000 |
| 1.500 | 312.000 | -37.000 |
| 1.600 | 325.000 | -31.000 |
| 1.700 | 336.000 | -24.000 |
| 1.800 | 351.000 | -22.000 |
| 1.900 | 389.000 | -20.000 |
| 2.000 | 418.000 | -22.000 |
| 2.200 | 366.000 | -32.000 |
| 2.400 | 348.000 | -22.000 |
| 2.600 | 351.000 | -17.000 |
| 2.800 | 358.000 | -14.000 |
| 3.000 | 368.000 | -13.000 |
| 3.200 | 369.000 | -15.000 |
| 3.400 | 335.000 | -15.000 |
| 3.600 | 353.000 | -12.000 |
| 3.800 | 370.000 | -9.000 |
| 4.000 | 365.000 | -9.000 |
| 4.010 | 399.000 | -6.000 |
| 4.500 | 421.000 | -26.000 |
| 5.000 | 431.000 | -18.000 |
| 6.000 | 441.000 | -15.000 |
| 6.500 | 513.000 | -15.000 |
| 7.000 | 549.000 | -22.000 |
| 7.250 | 642.000 | -24.000 |
| 7.500 | 748.000 | -33.000 |
| 7.750 | 748.000 | -50.000 |
| 8.000 | 641.000 | -60.000 |
| 8.500 | 555.000 | -58.000 |
| 9.000 | 534.000 | -75.000 |
| 10.000 | 342.000 | -92.000 |
| 11.000 | 203.000 | -90.000 |
| 12.000 | 159.000 | -77.000 |
| 13.000 | 117.000 | -70.000 |
| 14.000 | 87.000 | -56.000 |
| 15.000 | 94.000 | -25.000 |
| 16.000 | 106.000 | -20.000 |
| 17.000 | 119.000 | -13.000 |
| 18.000 | 132.000 | -10.000 |
| 19.000 | 143.000 | -7.000 |
| 20.000 | 140.000 | -10.000 |

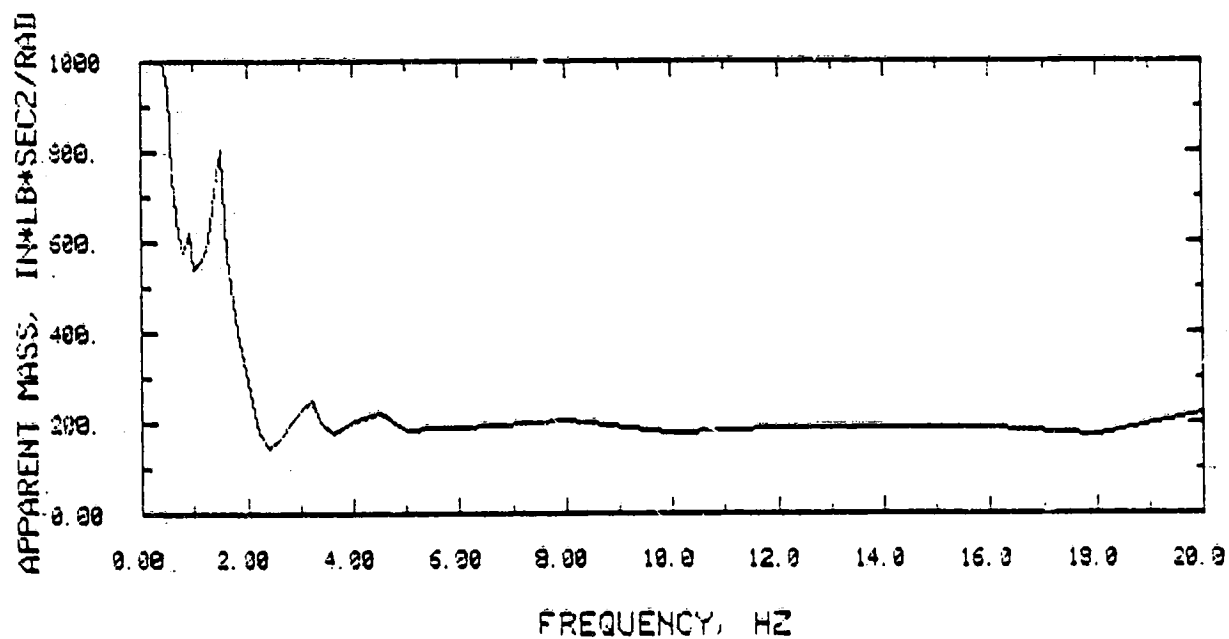
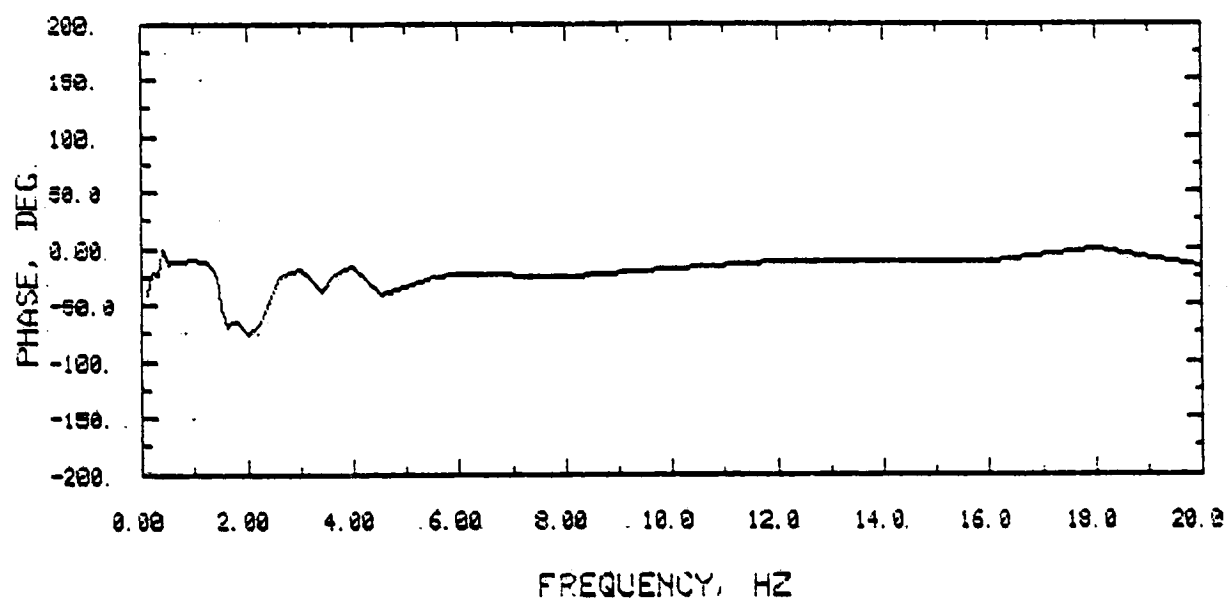
PLOT FIGURE : 89
CONFIGURATION : LIQUID OVER AIR ULLAGE
FILL / LIQUID : 54 GAL. / 2.52 SG ZNBR2
EXCITATION AMPLITUDE : 0.10 INCH P-P
EXCITATION AXIS-TYPE : X - ROTATION



PLOT FIGURE : B10
 CONFIGURATION : LIQUID OVER AIR ULLAGE
 FILL / LIQUID : 54 GAL. / 2.52 SG ZNBR2
 EXCITATION AMPLITUDE : 0.10 INCH P-P
 EXCITATION AXIS-TYPE : Y - ROTATION

| FREQUENCY | APPARENT MASS | PHASE |
|-----------|-----------------------------|---------|
| HZ | IN*LB*SEC ² /RAD | DEGREES |
| 0.100 | 10038.244 | -40.000 |
| 0.200 | 3346.081 | -22.000 |
| 0.300 | 2230.721 | -24.000 |
| 0.400 | 1045.650 | 0.000 |
| 0.500 | 936.903 | -15.000 |
| 0.600 | 743.574 | -13.000 |
| 0.700 | 614.586 | -13.000 |
| 0.800 | 575.108 | -12.000 |
| 0.900 | 619.645 | -10.000 |
| 1.000 | 535.373 | -11.000 |
| 1.100 | 553.071 | -13.000 |
| 1.200 | 580.917 | -13.000 |
| 1.300 | 633.578 | -17.000 |
| 1.400 | 751.161 | -25.000 |
| 1.500 | 803.060 | -52.000 |
| 1.600 | 562.037 | -70.000 |
| 1.700 | 497.860 | -66.000 |
| 1.800 | 402.769 | -66.000 |
| 2.000 | 284.417 | -77.000 |
| 2.200 | 172.835 | -68.000 |
| 2.400 | 139.420 | -45.000 |
| 2.600 | 163.344 | -25.000 |
| 2.800 | 196.326 | -20.000 |
| 3.000 | 226.790 | -19.000 |
| 3.200 | 245.074 | -27.000 |
| 3.400 | 196.828 | -38.000 |
| 3.600 | 172.984 | -26.000 |
| 3.800 | 185.378 | -19.000 |
| 4.000 | 204.947 | -17.000 |
| 4.500 | 218.167 | -40.000 |
| 5.000 | 181.806 | -34.000 |
| 5.500 | 183.945 | -25.000 |
| 6.000 | 183.945 | -24.000 |
| 8.000 | 203.195 | -25.000 |
| 10.000 | 175.389 | -18.000 |
| 12.000 | 183.945 | -13.000 |
| 14.000 | 186.084 | -12.000 |
| 16.000 | 186.084 | -12.000 |
| 18.000 | 168.973 | 0.000 |
| 20.000 | 218.167 | -17.000 |

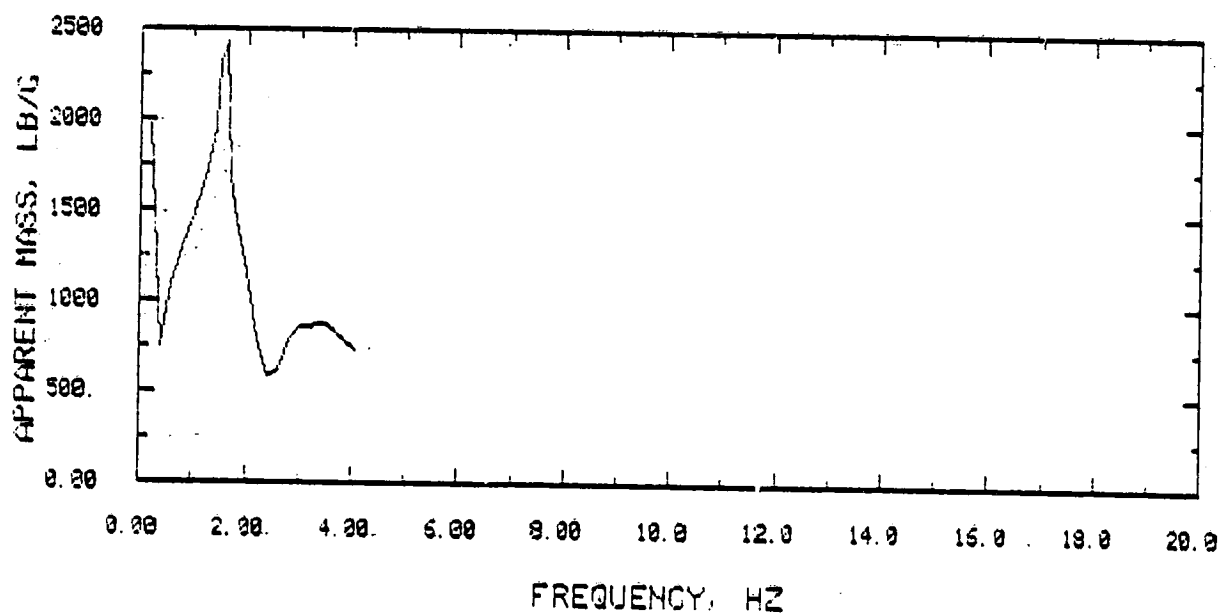
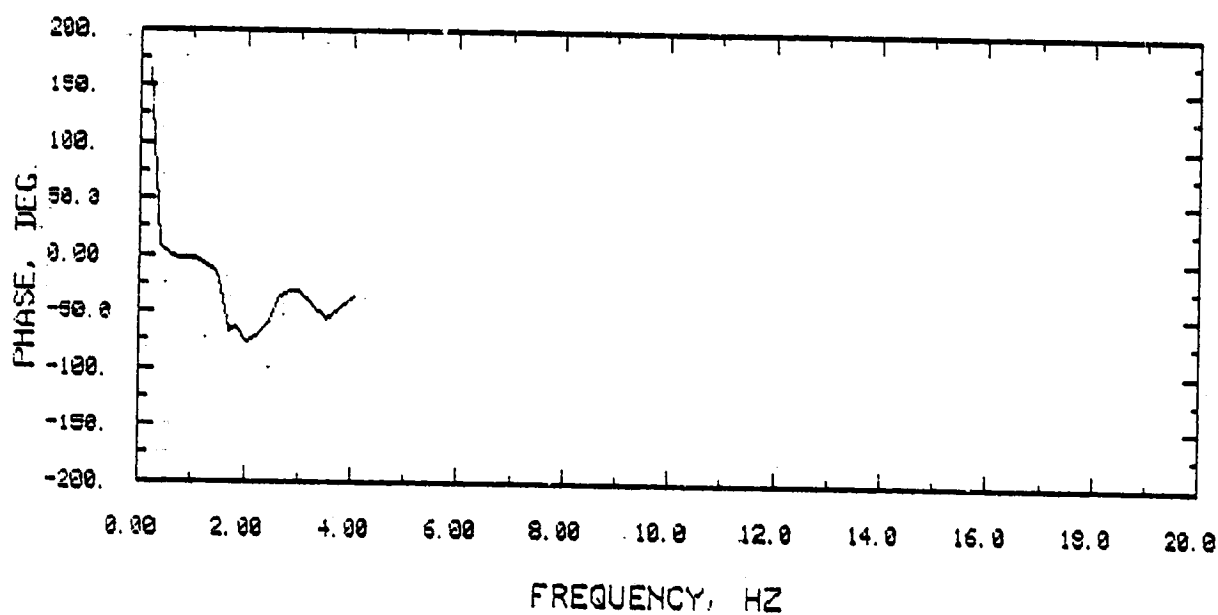
PLOT FIGURE : 810
CONFIGURATION : LIQUID OVER AIR ULLAGE
FILL / LIQUID : 54 GAL. / 2.52 SG ZNBR2
EXCITATION AMPLITUDE : 0.10 INCH P-P
EXCITATION AXIS-TYPE : Y - ROTATION



PLOT FIGURE : B11
 CONFIGURATION : LIQUID OVER AIR ULLAGE
 FILL / LIQUID : 54 GAL. / 2.52 SG ZNBR2
 EXCITATION AMPLITUDE : 0.05 INCH P-P
 EXCITATION AXIS-TYPE : X - TRANSLATION

| FREQUENCY HZ | APPARENT MASS LB/G | PHASE DEGREES |
|-----------------|-----------------------|------------------|
| 0.199 | 1967.000 | 164.000 |
| 0.399 | 737.000 | 8.000 |
| 0.600 | 1087.000 | -1.000 |
| 0.799 | 1287.000 | -4.000 |
| 1.001 | 1480.000 | -4.000 |
| 1.204 | 1676.000 | -7.000 |
| 1.402 | 1958.000 | -15.000 |
| 1.502 | 2318.000 | -23.000 |
| 1.608 | 2425.000 | -51.000 |
| 1.706 | 1690.000 | -67.000 |
| 1.809 | 1424.000 | -63.000 |
| 2.010 | 1145.000 | -77.000 |
| 2.210 | 789.000 | -73.000 |
| 2.410 | 576.000 | -60.000 |
| 2.620 | 617.000 | -38.000 |
| 2.820 | 782.000 | -32.000 |
| 3.020 | 857.000 | -32.000 |
| 3.510 | 862.000 | -56.000 |
| 4.020 | 735.000 | -37.000 |

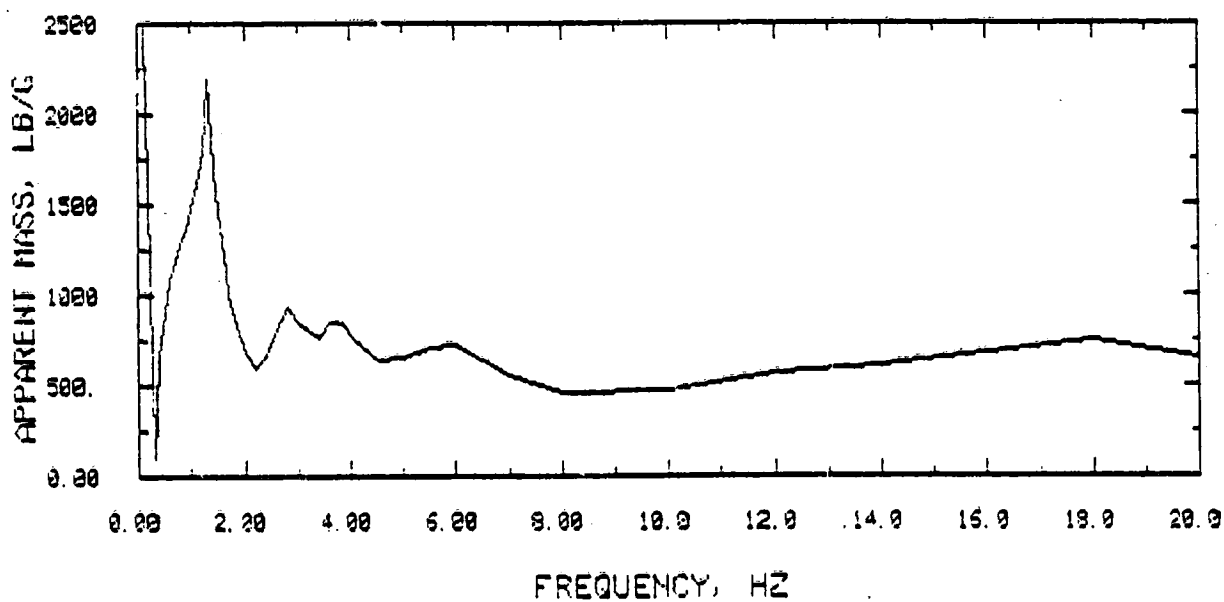
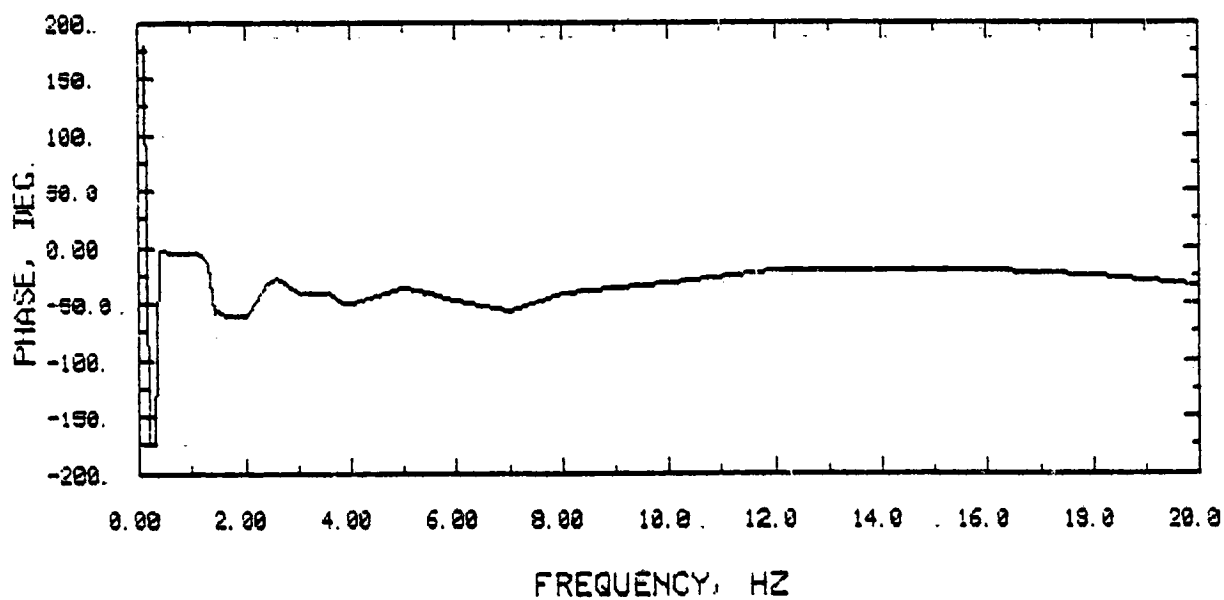
PLOT FIGURE : 911
CONFIGURATION : LIQUID OVER AIR ULLAGE
FILL / LIQUID : 54 GAL. / 2.52 SG ZNBR2
EXCITATION AMPLITUDE : 0.95 INCH P-P
EXCITATION AXIS-TYPE : X - TRANSLATION



PLOT FIGURE : B12
 CONFIGURATION : LIQUID OVER AIR ULLAGE
 FILL / LIQUID : 54 GAL. / 2.52 SQ ZNBR2
 EXCITATION AMPLITUDE : 0.25 INCH P-P
 EXCITATION AXIS-TYPE : X - TRANSLATION

| FREQUENCY HZ | APPARENT MASS LB/G | PHASE DEGREES |
|-----------------|-----------------------|------------------|
| 0.099 | 10294.000 | 178.000 |
| 0.199 | 1580.161 | -175.000 |
| 0.298 | 88.082 | -174.000 |
| 0.398 | 641.940 | -2.500 |
| 0.498 | 914.654 | -4.000 |
| 0.597 | 1075.387 | -4.500 |
| 0.698 | 1188.060 | -5.000 |
| 0.799 | 1274.258 | -5.000 |
| 0.900 | 1371.263 | -5.000 |
| 1.000 | 1478.357 | -5.000 |
| 1.100 | 1609.650 | -6.000 |
| 1.200 | 1797.973 | -9.000 |
| 1.302 | 2191.741 | -14.500 |
| 1.403 | 1688.850 | -58.000 |
| 1.504 | 1450.000 | -57.000 |
| 1.605 | 1211.547 | -61.000 |
| 1.706 | 986.338 | -62.000 |
| 1.807 | 874.368 | -61.000 |
| 1.909 | 783.427 | -62.000 |
| 2.008 | 667.342 | -60.000 |
| 2.210 | 584.556 | -47.000 |
| 2.410 | 659.902 | -33.000 |
| 2.610 | 817.554 | -28.000 |
| 2.810 | 927.216 | -34.000 |
| 3.010 | 844.352 | -40.000 |
| 3.210 | 800.107 | -42.000 |
| 3.410 | 762.147 | -42.000 |
| 3.610 | 843.295 | -42.000 |
| 3.820 | 836.210 | -50.000 |
| 4.020 | 757.000 | -51.000 |
| 4.500 | 635.000 | -43.000 |
| 5.000 | 648.000 | -37.000 |
| 5.500 | 702.000 | -41.000 |
| 6.000 | 710.000 | -47.000 |
| 7.000 | 550.000 | -56.000 |
| 8.000 | 445.000 | -40.500 |
| 10.000 | 470.000 | -33.000 |
| 12.000 | 560.000 | -20.500 |
| 14.000 | 605.000 | -20.000 |
| 16.000 | 670.000 | -22.000 |
| 18.000 | 745.000 | -25.000 |
| 20.000 | 651.000 | -34.000 |

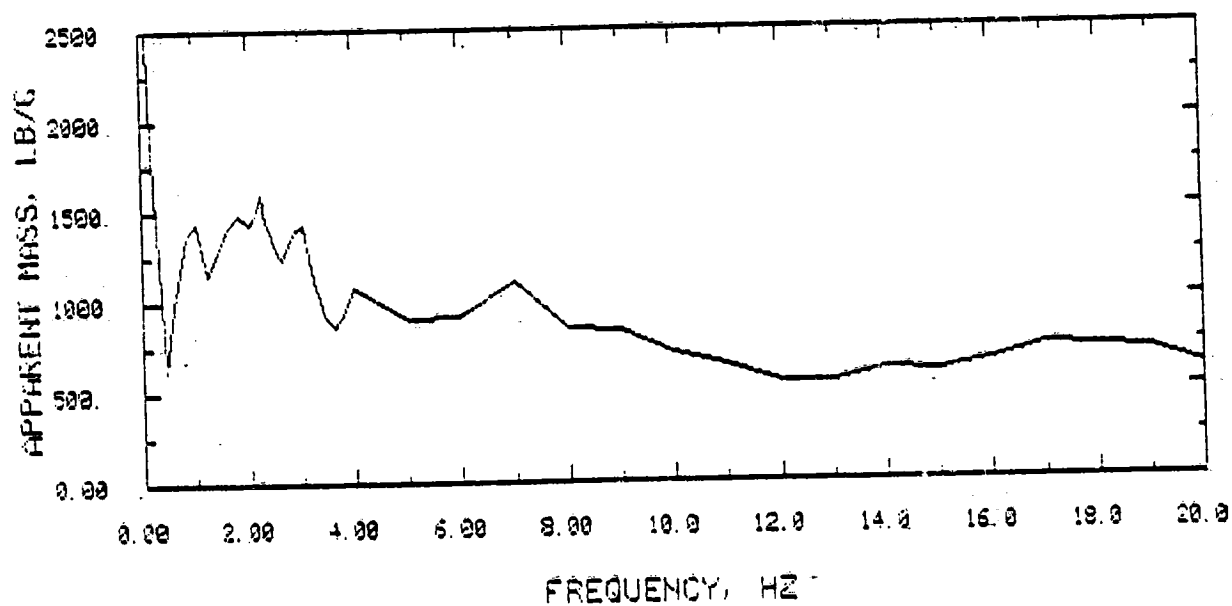
PLOT FIGURE : 812
CONFIGURATION : LIQUID OVER AIR ULLAGE
FILL / LIQUID : 54 GAL. / 2.52 SG ZNBR2
EXCITATION AMPLITUDE : 0.25 INCH P-P
EXCITATION AXIS-TYPE : X - TRANSLATION



PLOT FIGURE : B13
 CONFIGURATION : LIQUID OVER AIR ULLAGE
 FILL / LIQUID : 54 GAL. / 2.52 SQ ZNBR2
 EXCITATION AMPLITUDE : 0.25 INCH P-P
 EXCITATION AXIS-TYPE : Y - TRANSLATION

| FREQUENCY HZ | APPARENT MASS LB/G. |
|-----------------|------------------------|
| 0.103 | 10282.000 |
| 0.202 | 1821.000 |
| 0.413 | 618.000 |
| 0.598 | 985.000 |
| 0.818 | 1375.000 |
| 1.016 | 1439.000 |
| 1.217 | 1150.000 |
| 1.408 | 1281.000 |
| 1.629 | 1415.000 |
| 1.821 | 1473.000 |
| 2.028 | 1426.000 |
| 2.141 | 1476.000 |
| 2.232 | 1586.000 |
| 2.336 | 1433.000 |
| 2.439 | 1348.000 |
| 2.558 | 1256.000 |
| 2.646 | 1229.000 |
| 2.841 | 1381.000 |
| 3.049 | 1430.000 |
| 3.236 | 1120.000 |
| 3.448 | 921.000 |
| 3.650 | 852.000 |
| 3.846 | 952.000 |
| 4.000 | 1076.000 |
| 5.000 | 890.000 |
| 6.000 | 910.000 |
| 7.000 | 1110.000 |
| 8.000 | 840.000 |
| 9.000 | 820.000 |
| 10.000 | 700.000 |
| 11.000 | 630.000 |
| 12.000 | 540.000 |
| 13.000 | 540.000 |
| 14.000 | 600.000 |
| 15.000 | 590.000 |
| 16.000 | 645.000 |
| 17.000 | 735.000 |
| 18.000 | 715.000 |
| 19.000 | 700.000 |
| 20.000 | 600.000 |

PLOT FIGURE : 813
CONFIGURATION : LIQUID OVER AIR ULLAGE
FILL / LIQUID : 54 GAL. / 2.52 SG ZNBR2
EXCITATION AMPLITUDE : 0.25 INCH P-P
EXCITATION AXIS-TYPE : Y - TRANSLATION

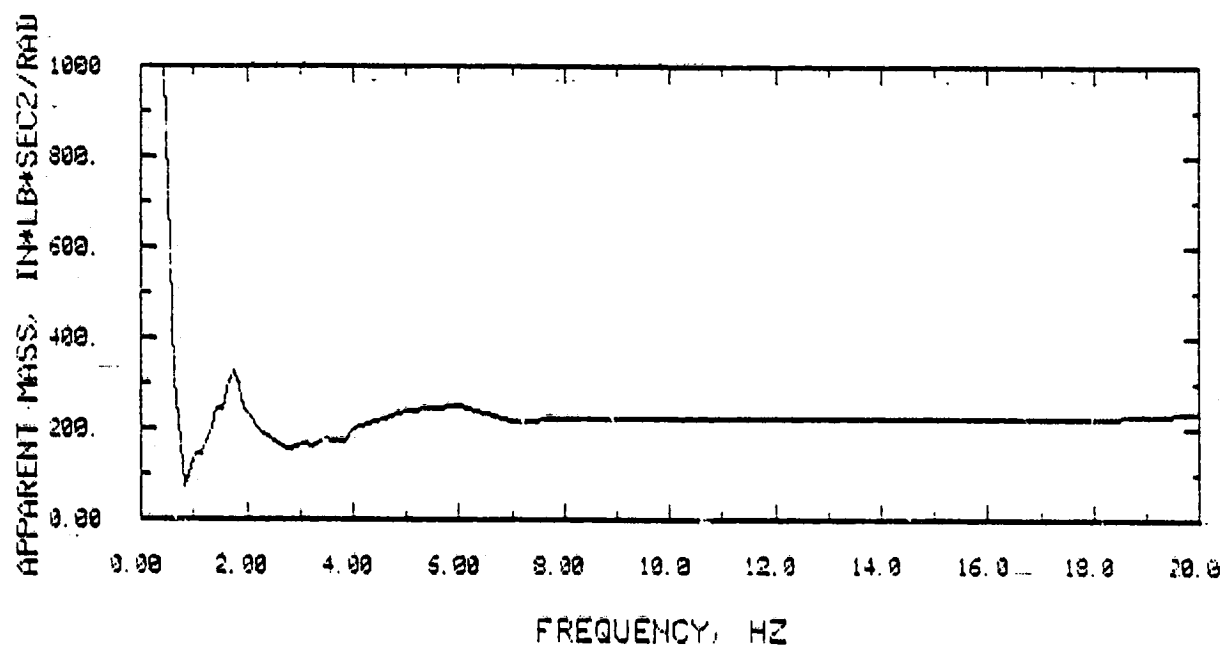


PLOT FIGURE
 CONFIGURATION
 FILL / LIQUID
 EXCITATION AMPLITUDE
 EXCITATION AXIS-TYPE

B14
 LIQUID OVER AIR ULLAGE
 54 GAL. / 2.52 SQ ZNBR2
 0.25 INCH P-P
 Z - ROTATION

| FREQUENCY HZ | APPARENT MASS IN*LB*SEC2/RAD |
|-----------------|---------------------------------|
| 0.106 | 17155.000 |
| 0.208 | 4893.000 |
| 0.417 | 1048.000 |
| 0.625 | 311.000 |
| 0.833 | 70.000 |
| 1.020 | 140.000 |
| 1.124 | 144.000 |
| 1.220 | 171.000 |
| 1.333 | 205.000 |
| 1.416 | 242.000 |
| 1.524 | 240.000 |
| 1.623 | 288.000 |
| 1.721 | 327.000 |
| 1.821 | 311.000 |
| 1.931 | 244.000 |
| 2.024 | 229.000 |
| 2.237 | 194.000 |
| 2.433 | 174.000 |
| 2.639 | 157.000 |
| 2.825 | 152.000 |
| 3.030 | 165.000 |
| 3.236 | 159.000 |
| 3.436 | 175.000 |
| 3.650 | 169.000 |
| 3.846 | 172.000 |
| 3.984 | 197.000 |
| 5.000 | 238.000 |
| 6.000 | 245.000 |
| 7.000 | 212.000 |
| 8.000 | 218.000 |
| 9.000 | 220.000 |
| 10.000 | 222.000 |
| 11.000 | 222.000 |
| 12.000 | 222.000 |
| 13.000 | 222.000 |
| 14.000 | 222.000 |
| 15.000 | 222.000 |
| 16.000 | 222.000 |
| 17.000 | 222.000 |
| 18.000 | 222.000 |
| 19.000 | 225.000 |
| 20.000 | 230.000 |

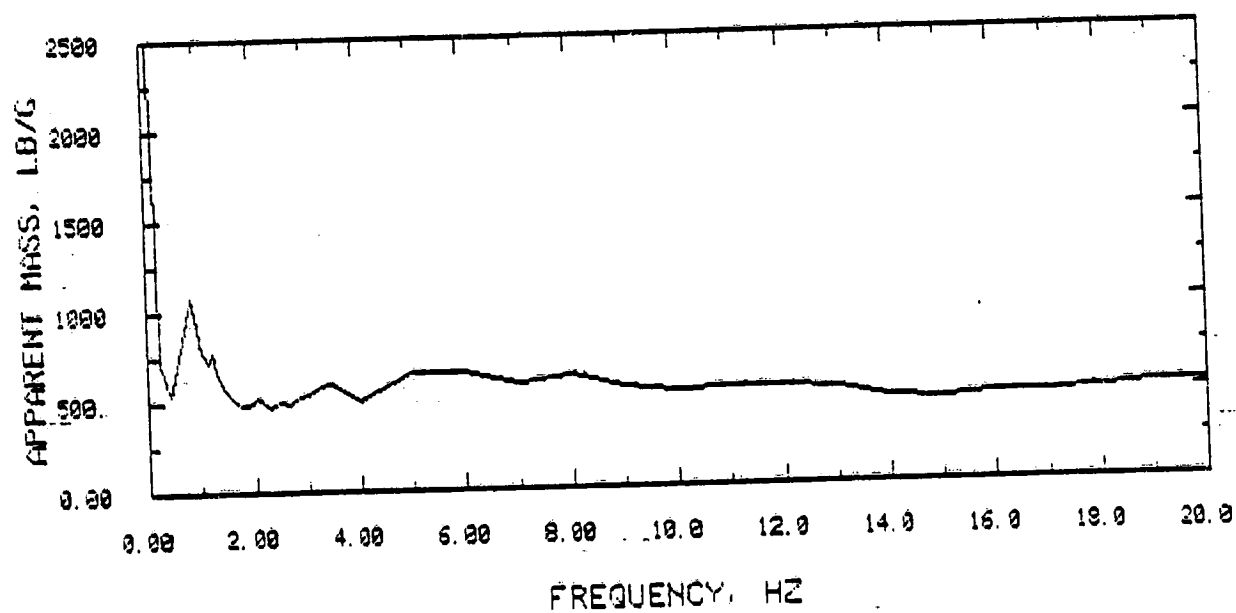
PLOT FIGURE : 914
CONFIGURATION : LIQUID-OVER AIR ULLAGE
FILL / LIQUID : 54 GAL. / 2.52-SG ZNBR2
EXCITATION AMPLITUDE : 0.25 INCH P-P
EXCITATION AXIS-TYPE : Z - ROTATION



PLOT FIGURE : B15
 CONFIGURATION : LIQUID OVER AIR ULLAGE
 FILL / LIQUID : 36 GAL. / 2.52 SG ZNBR2
 EXCITATION AMPLITUDE : 0.25 INCH P-P
 EXCITATION AXIS-TYPE : X - TRANSLATION

| FREQUENCY HZ | APPARENT MASS LB/G |
|-----------------|-----------------------|
| 0.104 | 5767.000 |
| 0.208 | 721.000 |
| 0.417 | 541.000 |
| 0.625 | 801.000 |
| 0.833 | 1070.000 |
| 0.926 | 912.000 |
| 1.031 | 773.000 |
| 1.124 | 713.000 |
| 1.235 | 770.000 |
| 1.316 | 678.000 |
| 1.429 | 575.000 |
| 1.540 | 529.000 |
| 1.639 | 495.000 |
| 1.754 | 483.000 |
| 1.852 | 479.000 |
| 1.961 | 488.000 |
| 2.041 | 526.000 |
| 2.273 | 469.000 |
| 2.500 | 488.000 |
| 2.703 | 482.000 |
| 2.857 | 527.000 |
| 3.030 | 532.000 |
| 3.236 | 579.000 |
| 3.448 | 592.000 |
| 3.704 | 542.000 |
| 4.000 | 489.000 |
| 5.000 | 640.000 |
| 6.000 | 650.000 |
| 7.000 | 580.000 |
| 8.000 | 620.000 |
| 9.000 | 550.000 |
| 10.000 | 520.000 |
| 11.000 | 530.000 |
| 12.000 | 530.000 |
| 13.000 | 525.000 |
| 14.000 | 465.000 |
| 15.000 | 450.000 |
| 16.000 | 475.000 |
| 17.000 | 485.000 |
| 18.000 | 500.000 |
| 19.000 | 520.000 |
| 20.000 | 520.000 |

PLOT FIGURE : 915
CONFIGURATION : LIQUID OVER AIR ULLAGE
FILL / LIQUID : 36 GAL. / 2.52 SG ZNBR2
EXCITATION AMPLITUDE : 0.25 INCH P-P
EXCITATION AXIS-TYPE : X - TRANSLATION

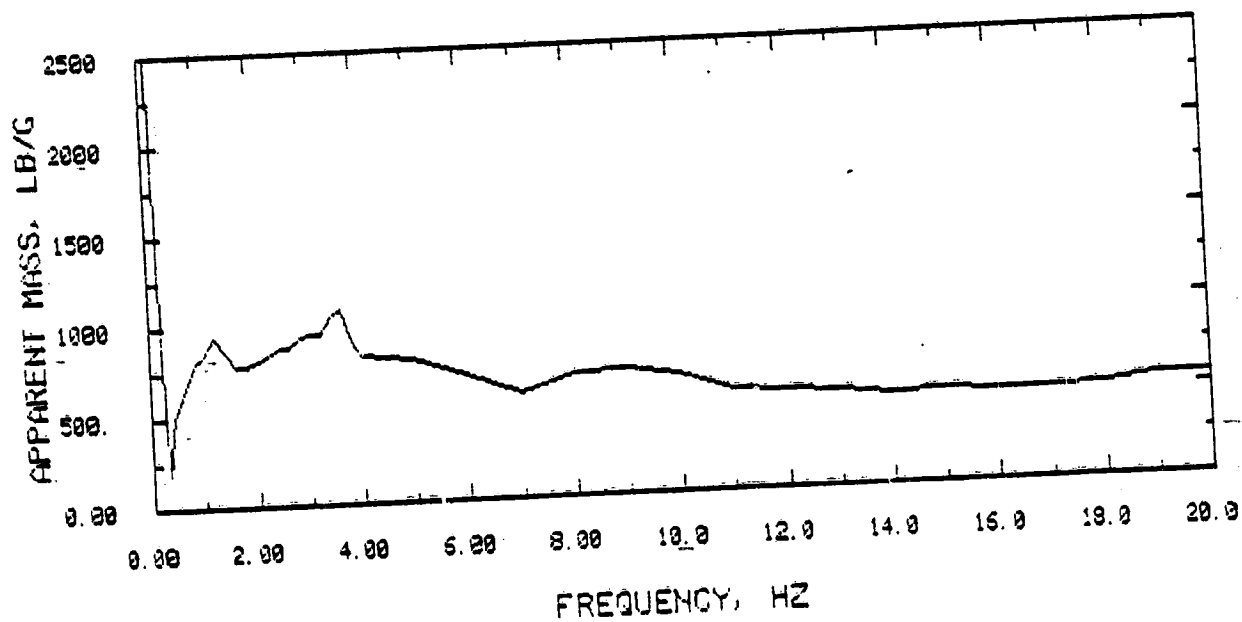


C-2

| | | |
|----------------------|---|-------------------------|
| PLOT FIGURE | : | B16 |
| CONFIGURATION | : | LIQUID OVER AIR ULLAGE |
| FILL / LIQUID | : | 36 GAL. / 2.52 SQ INBR2 |
| EXCITATION AMPLITUDE | : | 0.25 INCH P-P |
| EXCITATION AXIS-TYPE | : | Y- TRANSLATION |

| FREQUENCY | APPARENT MASS |
|-----------|---------------|
| HZ | LB/G |
| 0.103 | 6610.000 |
| 0.211 | 879.000 |
| 0.287 | 189.000 |
| 0.410 | 512.000 |
| 0.604 | 644.000 |
| 0.818 | 790.000 |
| 1.010 | 845.000 |
| 1.224 | 940.000 |
| 1.422 | 860.000 |
| 1.623 | 764.000 |
| 1.821 | 766.000 |
| 2.037 | 792.000 |
| 2.237 | 821.000 |
| 2.469 | 866.000 |
| 2.653 | 862.000 |
| 2.841 | 921.000 |
| 3.040 | 931.000 |
| 3.236 | 934.000 |
| 3.448 | 1036.000 |
| 3.650 | 1072.000 |
| 3.846 | 899.000 |
| 4.000 | 819.000 |
| 5.000 | 790.000 |
| 6.000 | 690.000 |
| 7.000 | 570.000 |
| 8.000 | 675.000 |
| 9.000 | 690.000 |
| 10.000 | 650.000 |
| 11.000 | 550.000 |
| 12.000 | 540.000 |
| 13.000 | 525.000 |
| 14.000 | 500.000 |
| 15.000 | 505.000 |
| 16.000 | 500.000 |
| 17.000 | 500.000 |
| 18.000 | 505.000 |
| 19.000 | 545.000 |
| 20.000 | 545.000 |

| | |
|----------------------|-------------------------|
| PLOT FIGURE | B16 |
| CONFIGURATION | LIQUID OVER AIR ULLAGE |
| FILL / LIQUID | 36 GAL. / 2.52 SG ZNBR2 |
| EXCITATION AMPLITUDE | 0.25 INCH P-P |
| EXCITATION AXIS-TYPE | Y - TRANSLATION |

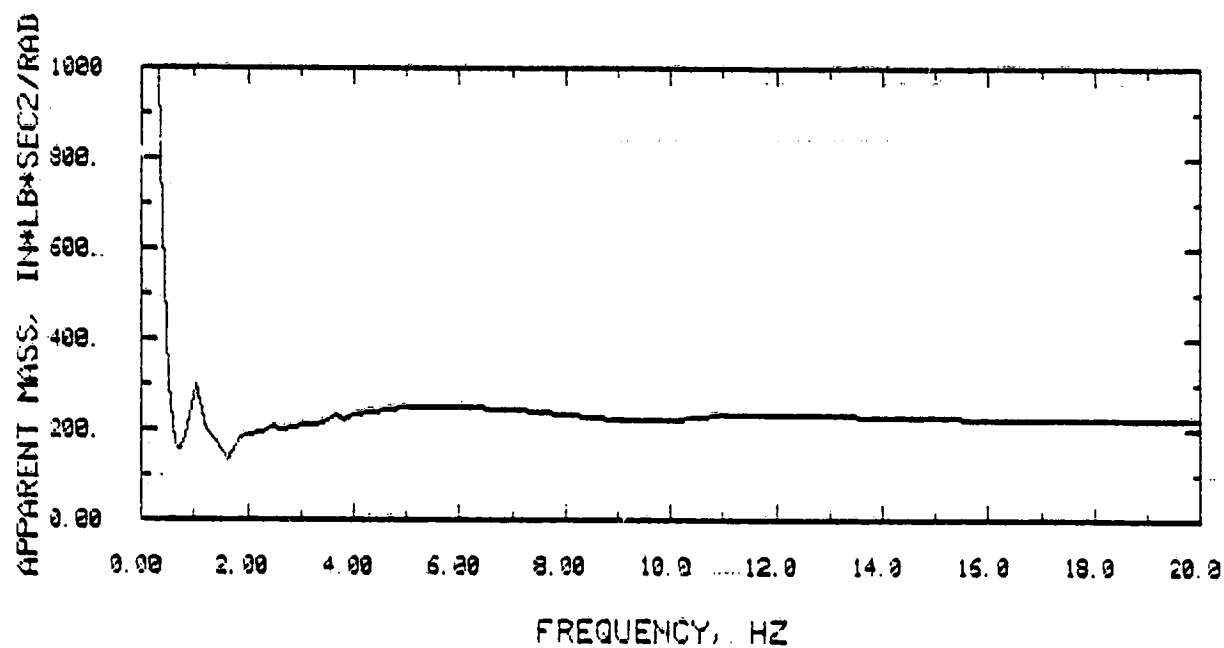


| | | |
|----------------------|---|-------------------------|
| PLOT FIGURE | : | B17 |
| CONFIGURATION | : | LIQUID OVER AIR ULLAGE |
| FILL / LIQUID | : | 36 GAL. / 2.52 SQ INBR2 |
| EXCITATION AMPLITUDE | : | 0.25 INCH P-P |
| EXCITATION AXIS-TYPE | : | Z - ROTATION |

| FREQUENCY | APPARENT MASS |
|-----------|-----------------------------|
| HZ | IN*LB*SEC ² /RAD |

| | |
|--------|-----------|
| 0.103 | 14330.000 |
| 0.203 | 3392.000 |
| 0.314 | 1227.000 |
| 0.397 | 655.000 |
| 0.510 | 303.000 |
| 0.610 | 163.000 |
| 0.694 | 151.000 |
| 0.820 | 181.000 |
| 0.917 | 231.000 |
| 1.009 | 298.000 |
| 1.215 | 197.000 |
| 1.406 | 172.000 |
| 1.621 | 129.000 |
| 1.835 | 180.000 |
| 2.028 | 184.000 |
| 2.242 | 193.000 |
| 2.439 | 204.000 |
| 2.639 | 200.000 |
| 2.833 | 204.000 |
| 3.040 | 210.000 |
| 3.236 | 211.000 |
| 3.448 | 212.000 |
| 3.650 | 228.000 |
| 3.846 | 217.000 |
| 4.000 | 228.000 |
| 5.000 | 245.000 |
| 6.000 | 250.000 |
| 7.000 | 240.000 |
| 8.000 | 230.000 |
| 9.000 | 222.000 |
| 10.000 | 222.000 |
| 11.000 | 230.000 |
| 12.000 | 228.000 |
| 13.000 | 228.000 |
| 14.000 | 225.000 |
| 15.000 | 227.000 |
| 16.000 | 220.000 |
| 17.000 | 220.000 |
| 18.000 | 222.000 |
| 19.000 | 220.000 |
| 20.000 | 220.000 |

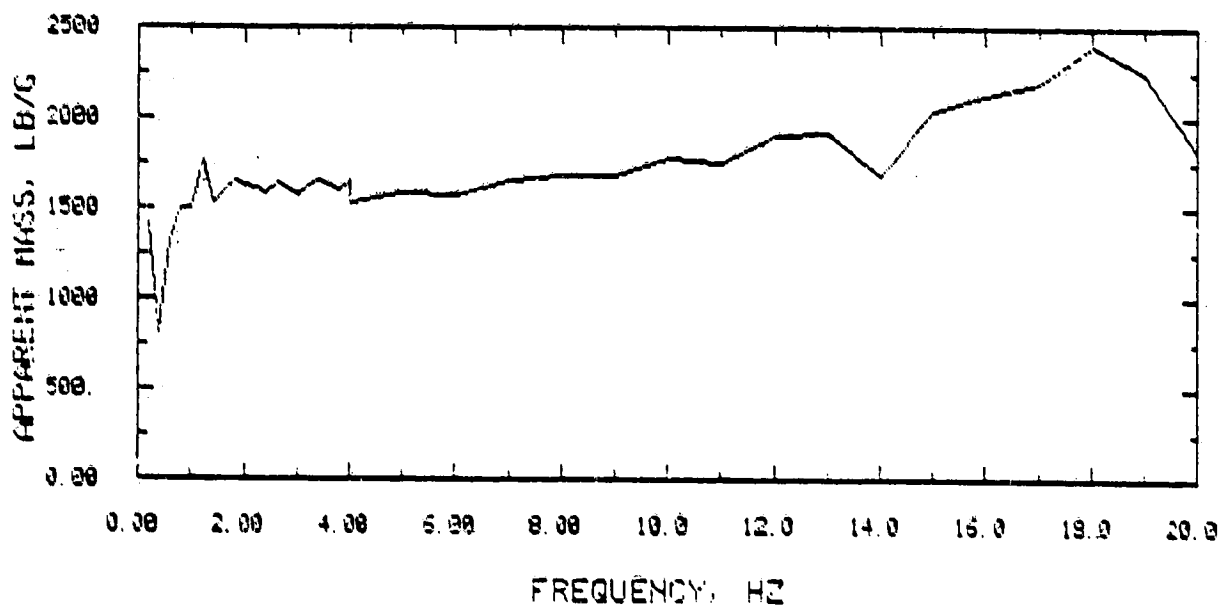
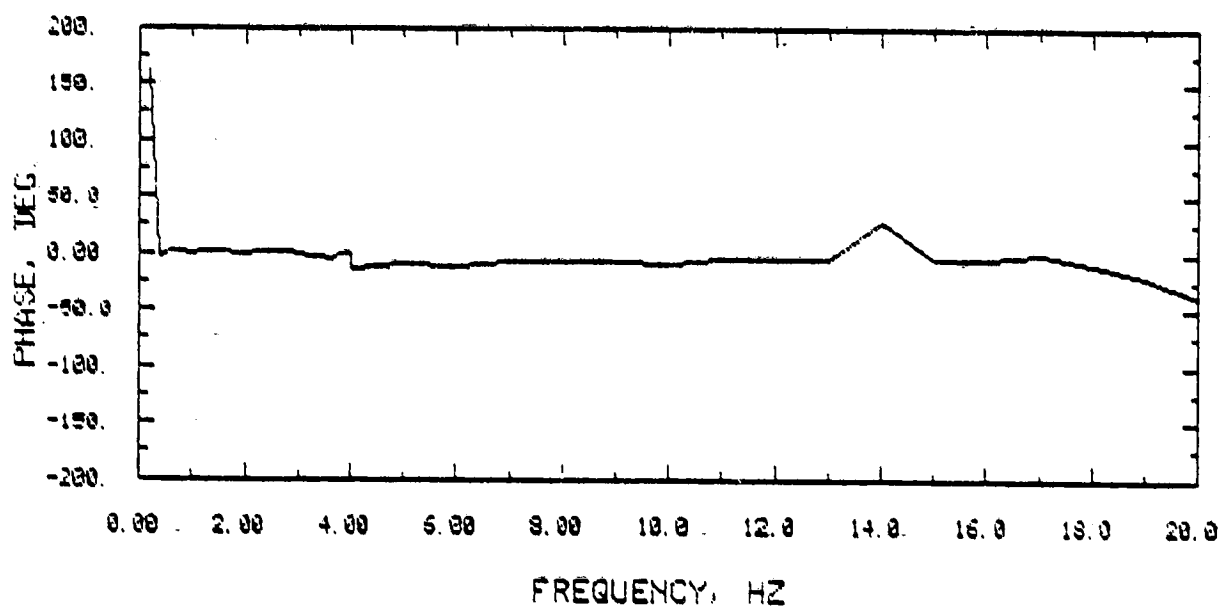
PLOT FIGURE : 817
CONFIGURATION : LIQUID OVER AIR ULLAGE
FILL / LIQUID : 36 GAL. / 2.52 SG ZNBR2
EXCITATION AMPLITUDE : 0.25 INCH P-P
EXCITATION AXIS-TYPE : Z - ROTATION



PLOT FIGURE : B18
 CONFIGURATION : LIQUID OVER WATER ULLAGE
 FILL / LIQUID : 54 GAL. / 2.0 SG ZNBR2
 EXCITATION AMPLITUDE : 0.25 INCH P-P
 EXCITATION AXIS-TYPE : X - TRANSLATION

| FREQUENCY HZ | APPARENT MASS.. LB/G | PHASE -- DEGREES |
|-----------------|-------------------------|---------------------|
| 0.200 | 1405.000 | 161.000 |
| 0.400 | 792.000 | -4.000 |
| 0.600 | 1311.000 | 2.000 |
| 0.800 | 1498.000 | 1.000 |
| 1.000 | 1496.000 | 0.000 |
| 1.200 | 1759.000 | 1.000 |
| 1.400 | 1520.000 | 1.000 |
| 1.600 | 1590.000 | 1.000 |
| 1.800 | 1646.000 | -1.000 |
| 2.000 | 1624.000 | -1.000 |
| 2.200 | 1601.000 | 1.000 |
| 2.400 | 1578.000 | 1.000 |
| 2.600 | 1626.000 | 2.000 |
| 2.800 | 1598.000 | 1.000 |
| 3.000 | 1557.000 | -2.000 |
| 3.200 | 1620.000 | -4.000 |
| 3.400 | 1639.000 | -3.000 |
| 3.600 | 1615.000 | -6.000 |
| 3.800 | 1595.000 | -2.000 |
| 4.000 | 1650.000 | 0.000 |
| 4.010 | 1525.000 | -15.500 |
| 5.000 | 1578.000 | -11.000 |
| 6.000 | 1560.000 | -11.500 |
| 7.000 | 1650.000 | -7.500 |
| 8.000 | 1670.000 | -7.000 |
| 9.000 | 1675.000 | -7.000 |
| 10.000 | 1770.000 | -9.000 |
| 11.000 | 1750.000 | -6.000 |
| 12.000 | 1900.000 | -6.500 |
| 13.000 | 1910.000 | -5.500 |
| 14.000 | 1675.000 | 28.500 |
| 15.000 | 2036.000 | -5.000 |
| 16.000 | 2120.000 | -6.000 |
| 17.000 | 2185.000 | 0.000 |
| 18.000 | 2400.000 | -10.500 |
| 19.000 | 2250.000 | -21.000 |
| 20.000 | 1819.000 | -38.000 |

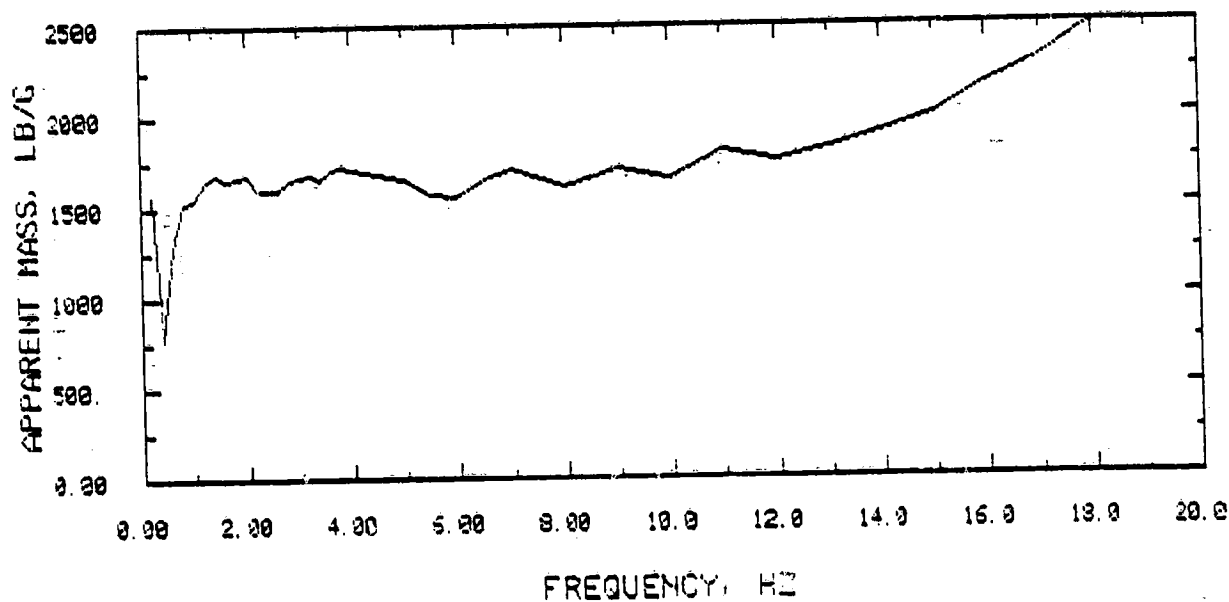
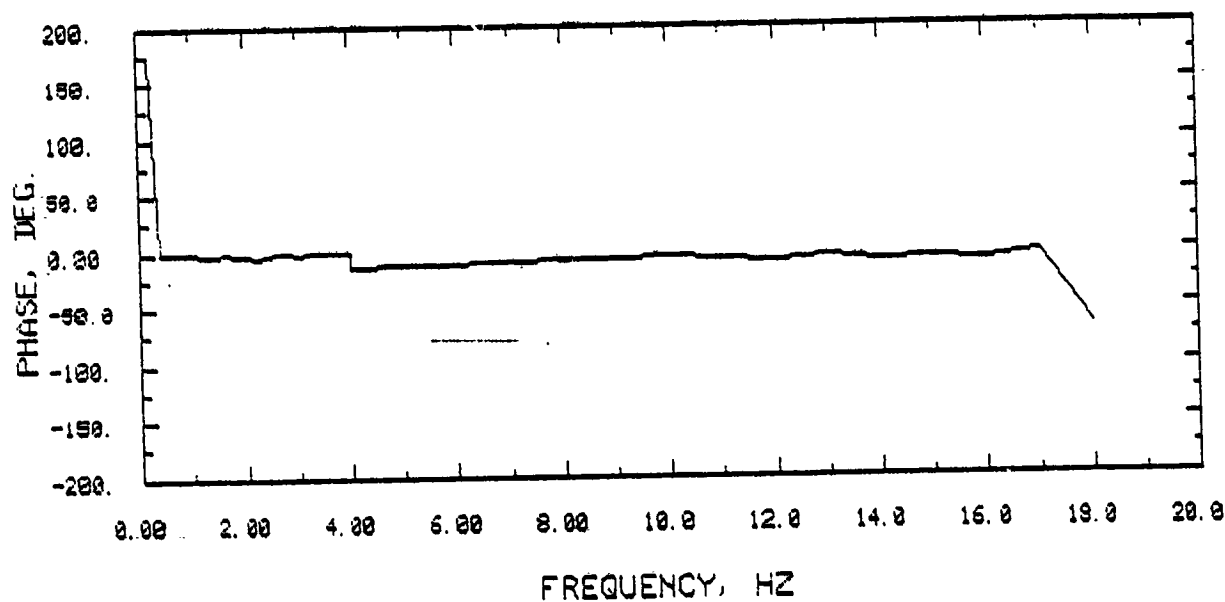
| | |
|----------------------|--------------------------|
| PLOT FIGURE | 818 |
| CONFIGURATION | LIQUID OVER WATER ULLAGE |
| FILL / LIQUID | 54 GAL. / 2.8 SG ZNBR2 |
| EXCITATION AMPLITUDE | 0.25 INCH P-P |
| EXCITATION AXIS-TYPE | X - TRANSLATION |



PLOT FIGURE : B19
 CONFIGURATION : LIQUID OVER WATER ULLAGE
 FILL / LIQUID : 54 GAL. / 2.0 SQ INBR2
 EXCITATION AMPLITUDE : 0.25 INCH P-P
 EXCITATION AXIS-TYPE : Y - TRANSLATION

| FREQUENCY HZ | APPARENT MASS LB/G | PHASE DEGREES |
|-----------------|-----------------------|------------------|
| 0.200 | 1565.000 | 174.000 |
| 0.400 | 776.600 | 0.000 |
| 0.600 | 1285.000 | -0.500 |
| 0.800 | 1516.000 | -2.000 |
| 1.000 | 1539.000 | -2.000 |
| 1.200 | 1639.000 | -4.000 |
| 1.400 | 1680.000 | -3.000 |
| 1.600 | 1651.000 | -2.000 |
| 1.800 | 1654.000 | -3.000 |
| 2.000 | 1669.000 | -4.000 |
| 2.200 | 1589.000 | -5.000 |
| 2.400 | 1591.000 | -4.000 |
| 2.600 | 1587.000 | -2.000 |
| 2.800 | 1640.000 | -2.000 |
| 3.000 | 1666.000 | -3.000 |
| 3.200 | 1669.000 | -1.000 |
| 3.400 | 1646.000 | -1.000 |
| 3.600 | 1703.000 | -1.000 |
| 3.800 | 1722.000 | -2.000 |
| 4.000 | 1697.000 | -2.000 |
| 4.010 | 1700.000 | -14.000 |
| 5.000 | 1650.000 | -13.000 |
| 5.500 | 1560.000 | -13.000 |
| 6.000 | 1550.000 | -13.000 |
| 6.500 | 1650.000 | -11.000 |
| 7.000 | 1700.000 | -9.000 |
| 8.000 | 1600.000 | -8.500 |
| 9.000 | 1700.000 | -8.000 |
| 10.000 | 1650.000 | -6.000 |
| 11.000 | 1800.000 | -7.000 |
| 12.000 | 1750.000 | -10.500 |
| 13.000 | 1809.000 | -6.000 |
| 14.000 | 1900.000 | -9.000 |
| 15.000 | 2000.000 | -8.000 |
| 16.000 | 2153.000 | -9.000 |
| 17.000 | 2292.000 | -3.000 |
| 18.000 | 2585.000 | -67.000 |

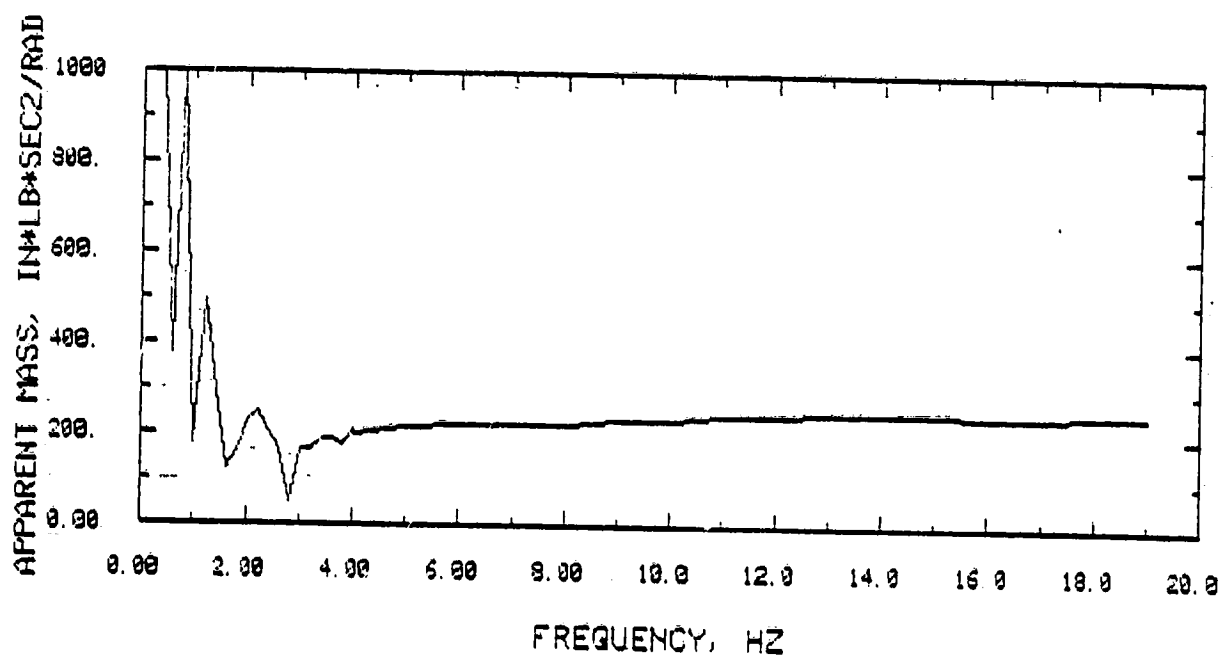
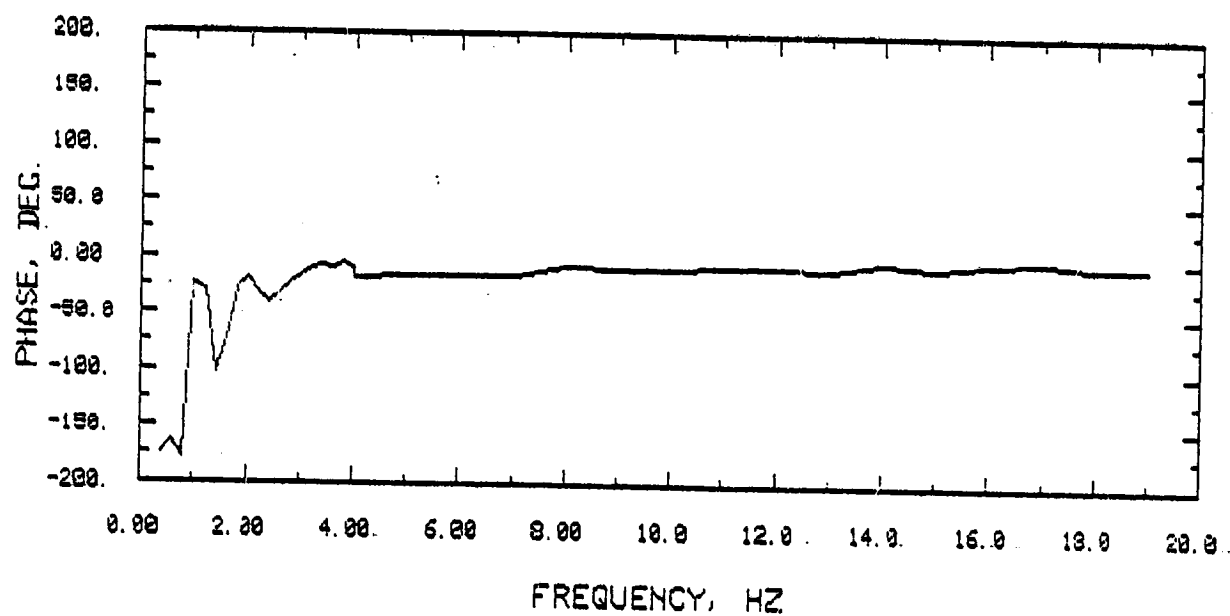
PLOT FIGURE : 819
CONFIGURATION : LIQUID OVER WATER ULLAGE
FILL / LIQUID : 54 GAL. / 2.0 SG ZNBR2
EXCITATION AMPLITUDE : 0.25 INCH P-P
EXCITATION AXIS-TYPE : Y - TRANSLATION



PLOT FIGURE : B20
 CONFIGURATION : LIQUID OVER WATER ULLAGE
 FILL / LIQUID : 54 GAL. / 2.0 SG ZNBR2
 EXCITATION AMPLITUDE : 0.25 INCH P-P
 EXCITATION AXIS-TYPE : Z - ROTATION

| FREQUENCY: | APPARENT MASS | PHASE |
|------------|-----------------------------|----------|
| HZ | IN*LB*SEC ² /RAD | DEGREES |
| 0.400 | 1425.000 | -175.000 |
| 0.600 | 374.700 | -164.000 |
| 0.800 | 1054.000 | -179.000 |
| 1.000 | 174.000 | -24.000 |
| 1.200 | 494.000 | -31.000 |
| 1.400 | 294.000 | -103.000 |
| 1.600 | 119.700 | -72.000 |
| 1.800 | 157.000 | -27.000 |
| 2.000 | 225.000 | -19.000 |
| 2.200 | 248.000 | -33.000 |
| 2.400 | 205.000 | -41.000 |
| 2.600 | 165.000 | -33.000 |
| 2.800 | 46.000 | -24.000 |
| 3.000 | 166.000 | -17.000 |
| 3.200 | 164.000 | -9.000 |
| 3.400 | 188.000 | -7.000 |
| 3.600 | 184.000 | -10.000 |
| 3.800 | 172.900 | -5.000 |
| 4.000 | 208.000 | -9.000 |
| 4.010 | 196.000 | -19.000 |
| 5.000 | 212.000 | -16.000 |
| 6.000 | 220.000 | -16.000 |
| 7.000 | 220.000 | -17.000 |
| 8.000 | 222.000 | -8.000 |
| 9.000 | 231.000 | -11.000 |
| 10.000 | 231.000 | -10.000 |
| 11.000 | 243.000 | -8.000 |
| 12.000 | 243.000 | -7.000 |
| 13.000 | 247.000 | -9.000 |
| 14.000 | 247.500 | -4.000 |
| 15.000 | 248.000 | -7.000 |
| 16.000 | 239.000 | -4.000 |
| 17.000 | 239.000 | -2.000 |
| 18.000 | 248.000 | -5.000 |
| 19.000 | 248.000 | -5.000 |

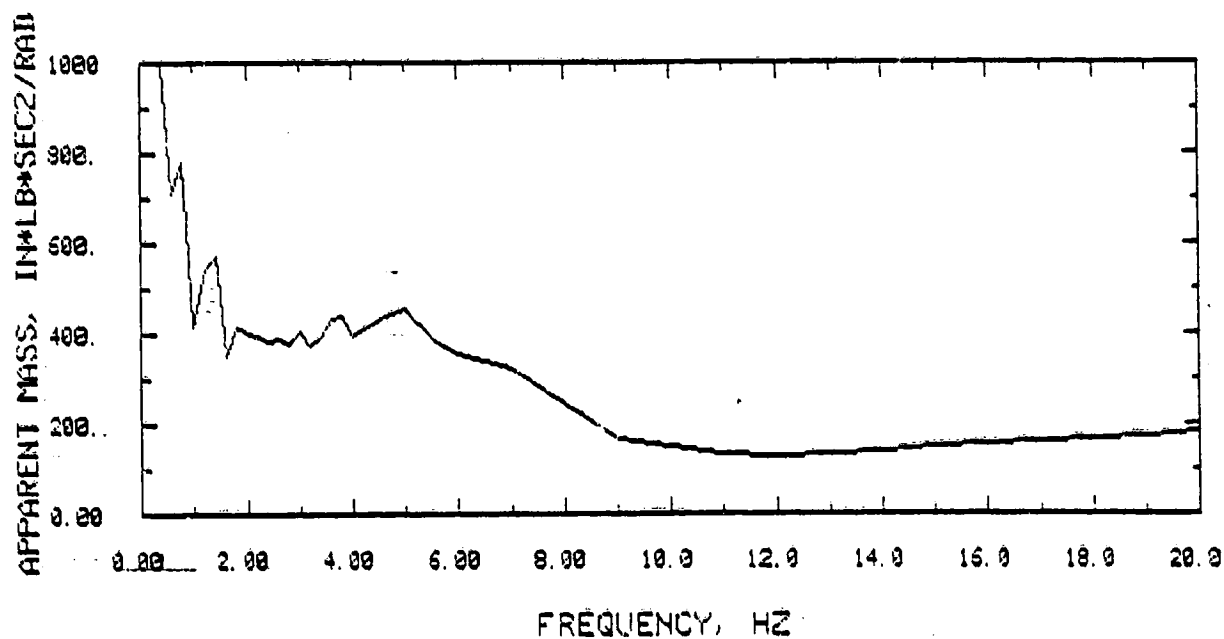
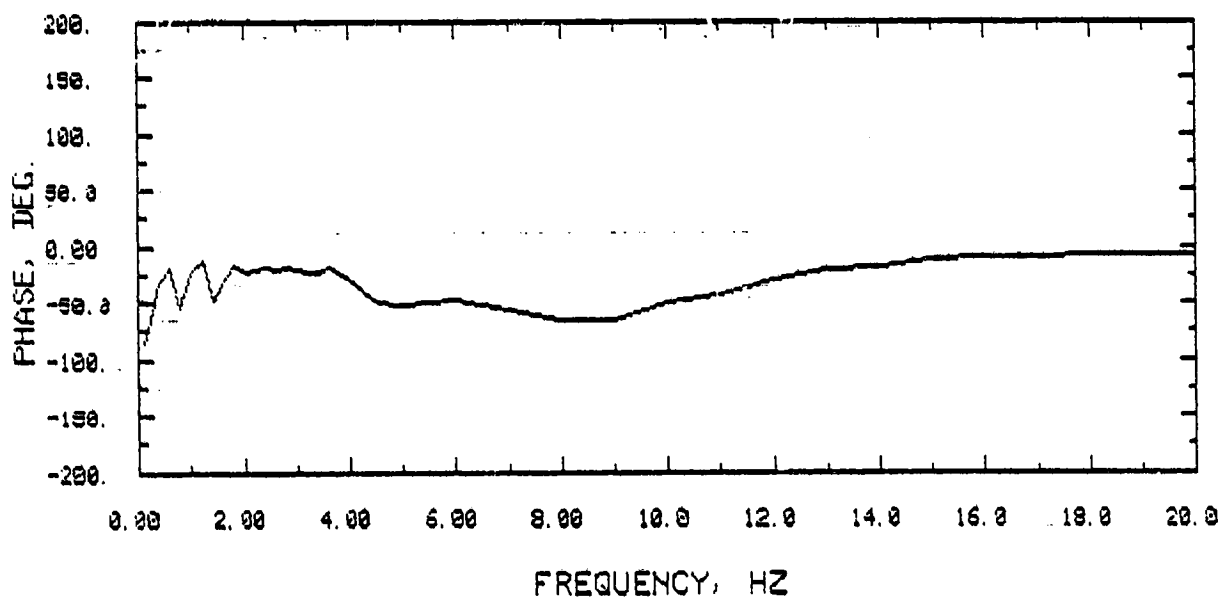
| | |
|----------------------|--------------------------|
| PLOT FIGURE | 820 |
| CONFIGURATION | LIQUID OVER WATER ULLAGE |
| FILL / LIQUID | 54 GAL. / 2.0 SG ZNBR2 |
| EXCITATION AMPLITUDE | 0.25 INCH P-P |
| EXCITATION AXIS-TYPE | 2 - ROTATION |



PLOT FIGURE : 821
 CONFIGURATION : LIQUID OVER WATER ULLAGE
 FILL / LIQUID : 54 GAL. / 2.0 SG ZNBR2
 EXCITATION AMPLITUDE : 0.25 INCH P-P
 EXCITATION AXIS-TYPE : X - ROTATION

| FREQUENCY HZ | APPARENT MASS IN*LB*SEC ² /RAD | PHASE DEGREES |
|-----------------|--|------------------|
| 0.100 | 6692.000 | -85.000 |
| 0.200 | 2288.000 | -73.000 |
| 0.400 | 1189.000 | -33.000 |
| 0.600 | 707.000 | -18.000 |
| 0.800 | 779.000 | -54.000 |
| 1.000 | 415.000 | -24.000 |
| 1.200 | 540.000 | -12.000 |
| 1.400 | 567.000 | -48.000 |
| 1.600 | 345.000 | -30.000 |
| 1.800 | 416.000 | -16.000 |
| 2.000 | 397.000 | -24.000 |
| 2.200 | 389.000 | -20.000 |
| 2.400 | 383.000 | -19.000 |
| 2.600 | 386.000 | -21.000 |
| 2.800 | 377.000 | -18.000 |
| 3.000 | 401.000 | -21.000 |
| 3.200 | 372.000 | -23.000 |
| 3.400 | 393.000 | -23.000 |
| 3.600 | 433.000 | -19.000 |
| 3.800 | 435.000 | -24.000 |
| 4.000 | 393.000 | -29.000 |
| 4.500 | 428.000 | -50.000 |
| 5.000 | 454.000 | -52.000 |
| 5.500 | 385.000 | -50.000 |
| 6.000 | 351.000 | -48.000 |
| 7.000 | 317.000 | -56.000 |
| 8.000 | 240.000 | -65.000 |
| 9.000 | 163.000 | -66.000 |
| 10.000 | 150.000 | -51.000 |
| 11.000 | 128.000 | -44.000 |
| 12.000 | 124.000 | -29.000 |
| 13.000 | 128.000 | -22.000 |
| 14.000 | 137.000 | -18.000 |
| 15.000 | 146.000 | -12.000 |
| 16.000 | 154.000 | -10.000 |
| 17.000 | 160.000 | -10.000 |
| 18.000 | 163.000 | -7.000 |
| 19.000 | 171.000 | -7.000 |
| 20.000 | 180.000 | -8.000 |

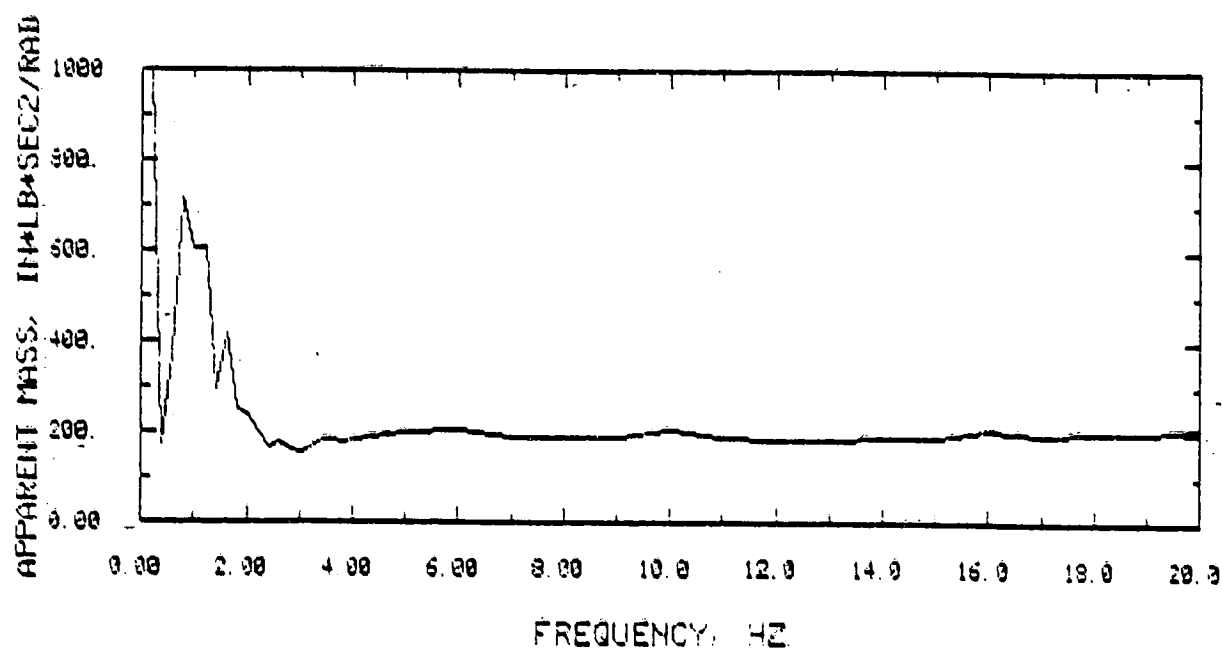
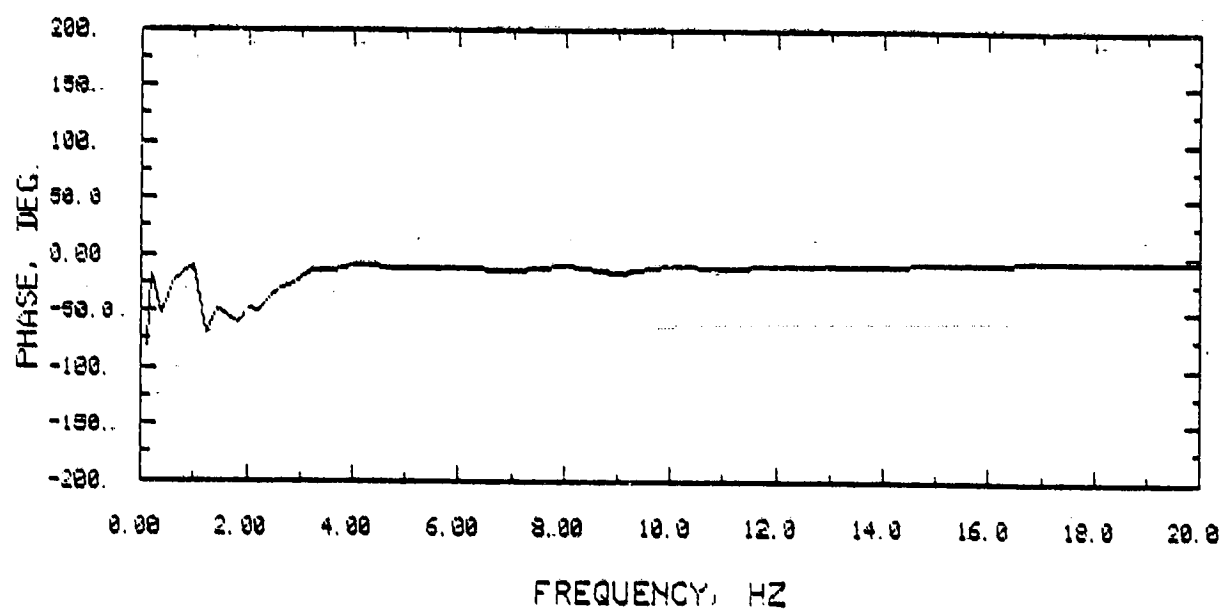
PLOT FIGURE : B21
CONFIGURATION : LIQUID OVER WATER ULLAGE
FILL / LIQUID : 54 GAL. / 2.0 SG ZNBR2
EXCITATION AMPLITUDE : 0.25 INCH P-P
EXCITATION AXIS-TYPE : X - ROTATION



PLOT FIGURE : 822
 CONFIGURATION : LIQUID OVER WATER ULLAGE
 FILL / LIQUID : 54 GAL. / 2.0 SG ZNBR2
 EXCITATION AMPLITUDE : 0.25 INCH P-P
 EXCITATION AXIS-TYPE : Y - ROTATION

| FREQUENCY HZ. | APPARENT MASS IN*LB*SEC ² /RAD | PHASE DEGREES |
|------------------|--|------------------|
| 0.100 | 12046.000 | -82.000 |
| 0.200 | 1003.000 | -16.000 |
| 0.400 | 171.300 | -53.000 |
| 0.600 | 372.000 | -26.000 |
| 0.800 | 715.000 | -17.000 |
| 1.000 | 602.000 | -11.000 |
| 1.200 | 605.000 | -69.000 |
| 1.400 | 293.000 | -47.000 |
| 1.600 | 413.000 | -55.000 |
| 1.800 | 247.000 | -60.000 |
| 2.000 | 236.000 | -48.000 |
| 2.200 | 195.000 | -51.000 |
| 2.400 | 162.000 | -38.000 |
| 2.600 | 176.000 | -31.000 |
| 2.800 | 157.300 | -27.000 |
| 3.000 | 155.000 | -21.000 |
| 3.200 | 163.000 | -15.000 |
| 3.400 | 180.000 | -14.000 |
| 3.600 | 181.000 | -14.000 |
| 3.800 | 176.000 | -12.000 |
| 4.000 | 183.000 | -9.000 |
| 5.000 | 197.000 | -12.000 |
| 6.000 | 205.000 | -13.000 |
| 7.000 | 188.000 | -14.000 |
| 8.000 | 188.000 | -11.000 |
| 9.000 | 188.000 | -16.000 |
| 10.000 | 205.000 | -10.000 |
| 11.000 | 188.000 | -13.000 |
| 12.000 | 180.000 | -11.000 |
| 13.000 | 180.000 | -9.000 |
| 14.000 | 184.000 | -9.000 |
| 15.000 | 188.000 | -7.000 |
| 16.000 | 205.000 | -7.000 |
| 17.000 | 194.000 | -6.000 |
| 18.000 | 197.000 | -6.000 |
| 19.000 | 197.000 | -5.000 |
| 20.000 | 207.000 | -6.000 |

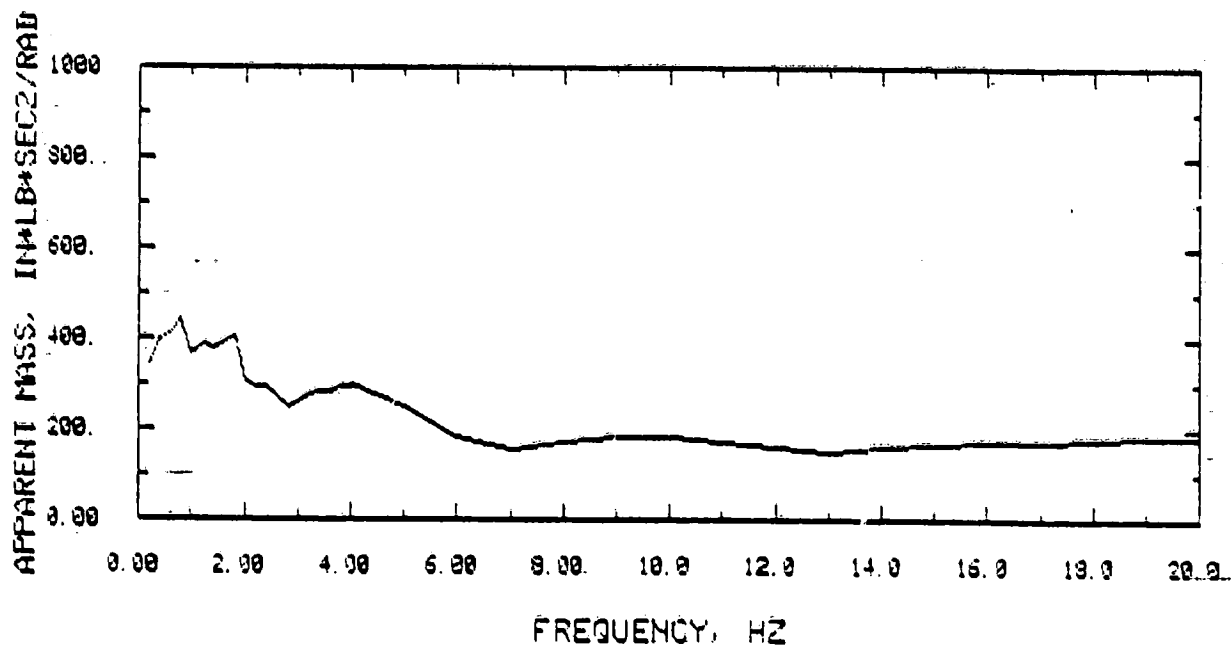
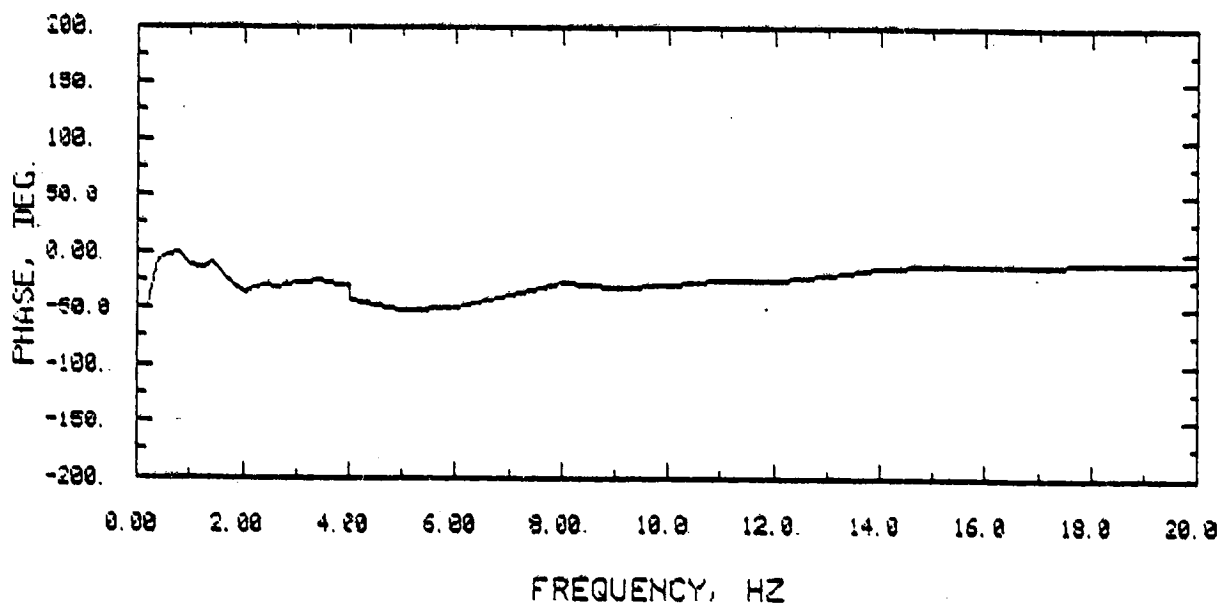
| | |
|----------------------|--------------------------|
| PLOT FIGURE | 822 |
| CONFIGURATION | LIQUID OVER WATER ULLAGE |
| FILL / LIQUID | 54 GAL. / 2.0 SG ZNBR2 |
| EXCITATION AMPLITUDE | 0.25 INCH P-P |
| EXCITATION AXIS-TYPE | Y - ROTATION |



PLOT FIGURE : 823
 CONFIGURATION : LIQUID OVER WATER ULLAGE
 FILL / LIQUID : 54 GAL. / 1.25 SQ INBR2
 EXCITATION AMPLITUDE : 1.0 INCH P-P
 EXCITATION AXIS-TYPE : X - ROTATION

| FREQUENCY | APPARENT MASS | PHASE |
|-----------|-----------------------------|---------|
| HZ | IN*LB*SEC ² /RAD | DEGREES |
| 0.200 | 343.000 | -47.000 |
| 0.400 | 400.000 | -8.000 |
| 0.600 | 409.000 | -4.000 |
| 0.800 | 440.000 | -1.000 |
| 1.000 | 364.000 | -13.000 |
| 1.200 | 384.000 | -14.000 |
| 1.400 | 377.000 | -10.000 |
| 1.600 | 392.000 | -20.000 |
| 1.800 | 403.000 | -29.000 |
| 2.000 | 301.000 | -37.000 |
| 2.200 | 294.000 | -32.000 |
| 2.400 | 290.000 | -31.000 |
| 2.600 | 270.000 | -33.000 |
| 2.800 | 250.000 | -31.000 |
| 3.000 | 258.000 | -28.000 |
| 3.200 | 276.000 | -27.000 |
| 3.400 | 281.000 | -26.000 |
| 3.600 | 281.000 | -27.000 |
| 3.800 | 291.000 | -29.000 |
| 4.000 | 291.000 | -29.000 |
| 4.010 | 300.000 | -43.000 |
| 5.000 | 248.000 | -53.000 |
| 6.000 | 180.000 | -50.000 |
| 7.000 | 154.000 | -39.000 |
| 8.000 | 171.000 | -28.000 |
| 9.000 | 180.000 | -32.000 |
| 10.000 | 180.000 | -31.000 |
| 11.000 | 171.000 | -26.000 |
| 12.000 | 160.000 | -26.000 |
| 13.000 | 150.000 | -20.000 |
| 14.000 | 156.000 | -15.000 |
| 15.000 | 163.000 | -13.000 |
| 16.000 | 169.000 | -12.000 |
| 17.000 | 171.000 | -12.000 |
| 18.000 | 176.000 | -11.000 |
| 19.000 | 179.000 | -9.000 |
| 20.000 | 182.000 | -9.000 |

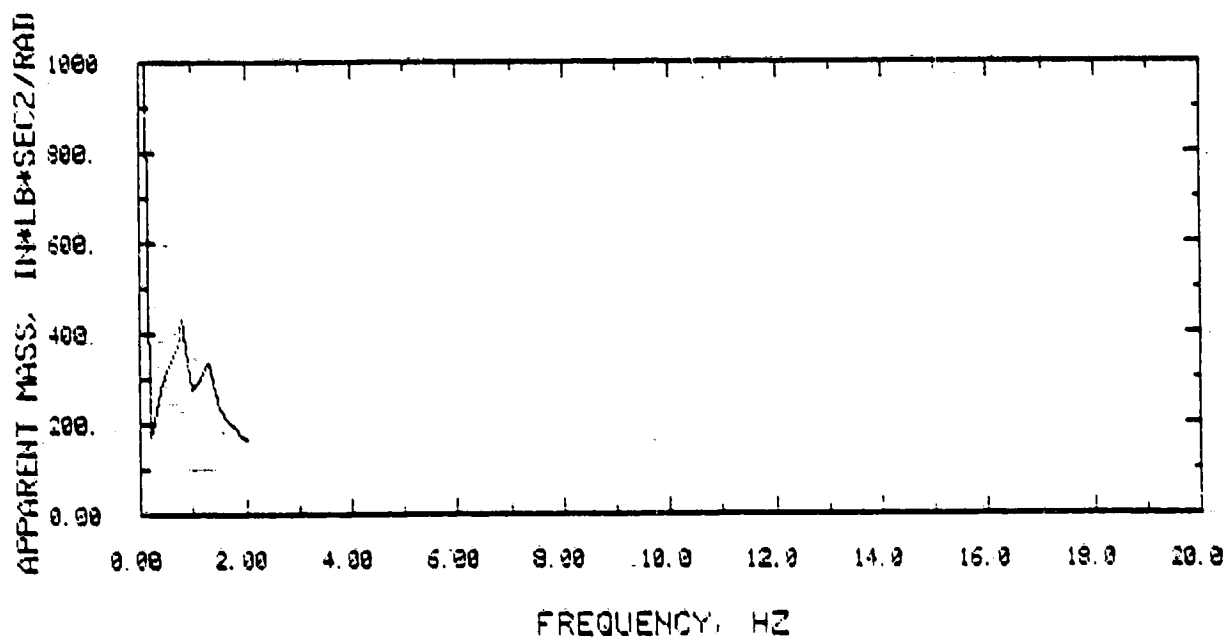
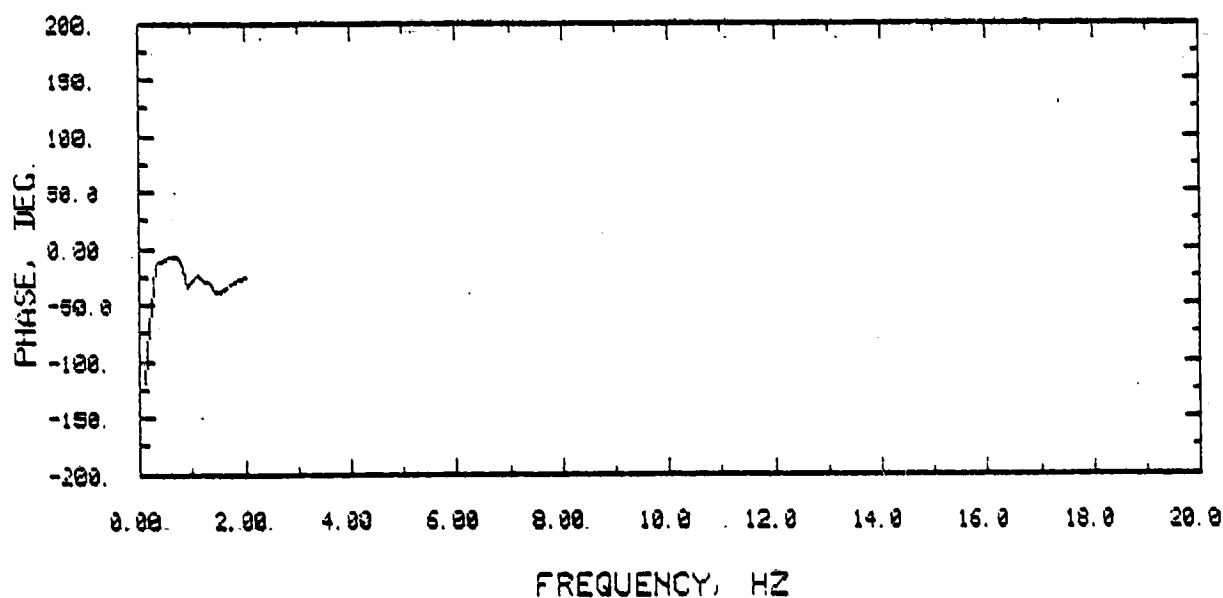
| | |
|----------------------|--------------------------|
| PLOT FIGURE | 823 |
| CONFIGURATION | LIQUID OVER WATER ULLAGE |
| FILL / LIQUID | 54 GAL. / 1.25 SG ZNBR2 |
| EXCITATION AMPLITUDE | 1.0 INCH P-P |
| EXCITATION AXIS-TYPE | X - ROTATION |



PLOT FIGURE : B24
 CONFIGURATION : LIQUID OVER WATER ULLAGE
 FILL / LIQUID : 54 GAL. / 1.25 SG ZNBR2
 EXCITATION AMPLITUDE : 1.0 INCH P-P
 EXCITATION AXIS-TYPE : Y - ROTATION

| FREQUENCY | APPARENT MASS | PHASE |
|-----------|-----------------------------|----------|
| HZ | IN*LB*SEC ² /RAD | DEGREES |
| 0.100 | 1003.000 | -119.000 |
| 0.200 | 167.400 | -68.000 |
| 0.300 | 223.000 | -15.000 |
| 0.400 | 273.000 | -12.000 |
| 0.500 | 321.000 | -9.000 |
| 0.600 | 344.000 | -7.000 |
| 0.700 | 367.000 | -7.000 |
| 0.800 | 433.000 | -12.000 |
| 0.900 | 329.000 | -35.000 |
| 1.000 | 277.000 | -30.000 |
| 1.100 | 292.000 | -23.000 |
| 1.200 | 324.000 | -30.000 |
| 1.300 | 330.000 | -31.000 |
| 1.400 | 268.000 | -38.000 |
| 1.500 | 233.000 | -39.000 |
| 1.600 | 211.000 | -37.000 |
| 1.700 | 199.000 | -33.000 |
| 1.800 | 185.000 | -31.000 |
| 1.900 | 171.000 | -28.000 |
| 2.000 | 163.000 | -25.000 |

PLOT FIGURE : 824
CONFIGURATION : LIQUID OVER WATER ULLAGE
FILL / LIQUID : 54 GAL. / 1.25 SG ZNBR2
EXCITATION AMPLITUDE : 1.0 INCH P-P
EXCITATION AXIS-TYPE : Y - ROTATION



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APPENDIX C
LOW GRAVITY VERIFICATION

APPENDIX C. LOW GRAVITY VERIFICATION

The aft tank slosh tests for which the ullage contained liquid can be rationalized with the other tests only if the liquid mass oscillating in the ullage is taken into account. Figure C.1 shows one model that can be used to aid in making such rationalizations. The model is meant to duplicate only the low-frequency slosh dynamics, and not the static stability, because these tests investigated only sloshing. For that reason, it is clearer to represent the sloshing of the simulated propellant (i.e., zinc bromide) by an ordinary pendulum rather than by an inverted pendulum attached to a spring and dashpot as was shown in Figure 1b. The damping is also neglected in this Appendix.

The liquid oscillating in the ullage is represented by a pendulum, in this case inverted to demonstrate that if the heavier liquid is placed in the ullage, the whole system becomes unstable. The rod connecting the two pendulums represents the restraint placed on the motion by the bladder separating the liquids. Since the liquid is incompressible and the total volume of the tank is constant, when the zinc bromide sloshes forward an equal volume of water must slosh backward. Further, the two volumes must move equal distances. Thus, $L_1\psi_1 = L_2\psi_2$ and $m_1/\rho_1 = m_2/\rho_2$.

The zinc bromide pendulum is attached to its pivot by a torsional spring K_1 that simulates the bladder stiffness. No such spring is required for the water pendulum, since all the stiffness is represented by K_1 . The gravitational part of the restoring force acting on the slosh mass m_1 is simply m_1gL . Comparing this representation to the alternative representation of the slosh spring constant derived in Section V, it is evident that $L_1 = 144/32.96 = 4.37$ inches and $K_1 = 9.54 L_1^2 = 182.1$ in-lb. (This transformation decomposes the slosh spring into an equivalent torsional spring and pendulum; the same natural frequency is predicted by both representations.)

If F_1 and F_2 are the forces exerted on the pendulum arms by the connecting rod, moment equilibrium about the central hinge requires that

$$F_2 = F_1 (d_1/d_2) \quad (C.1)$$

Using this relation, the equations of motion of the two slosh masses can be written as:

$$m_1 L_1^2 \ddot{\psi}_1 + (K_1 + m_1 L_1 g) \psi_1 = -F_1 a_1 - m_1 L_1 \ddot{x}_1 \quad (C.2)$$

and

$$m_2 L_2^2 \ddot{\psi}_2 - m_2 L_2 g \psi_2 = F_1 a_2 (d_2/d_1) + m_2 L_2 \ddot{x}_2 \quad (C.3)$$

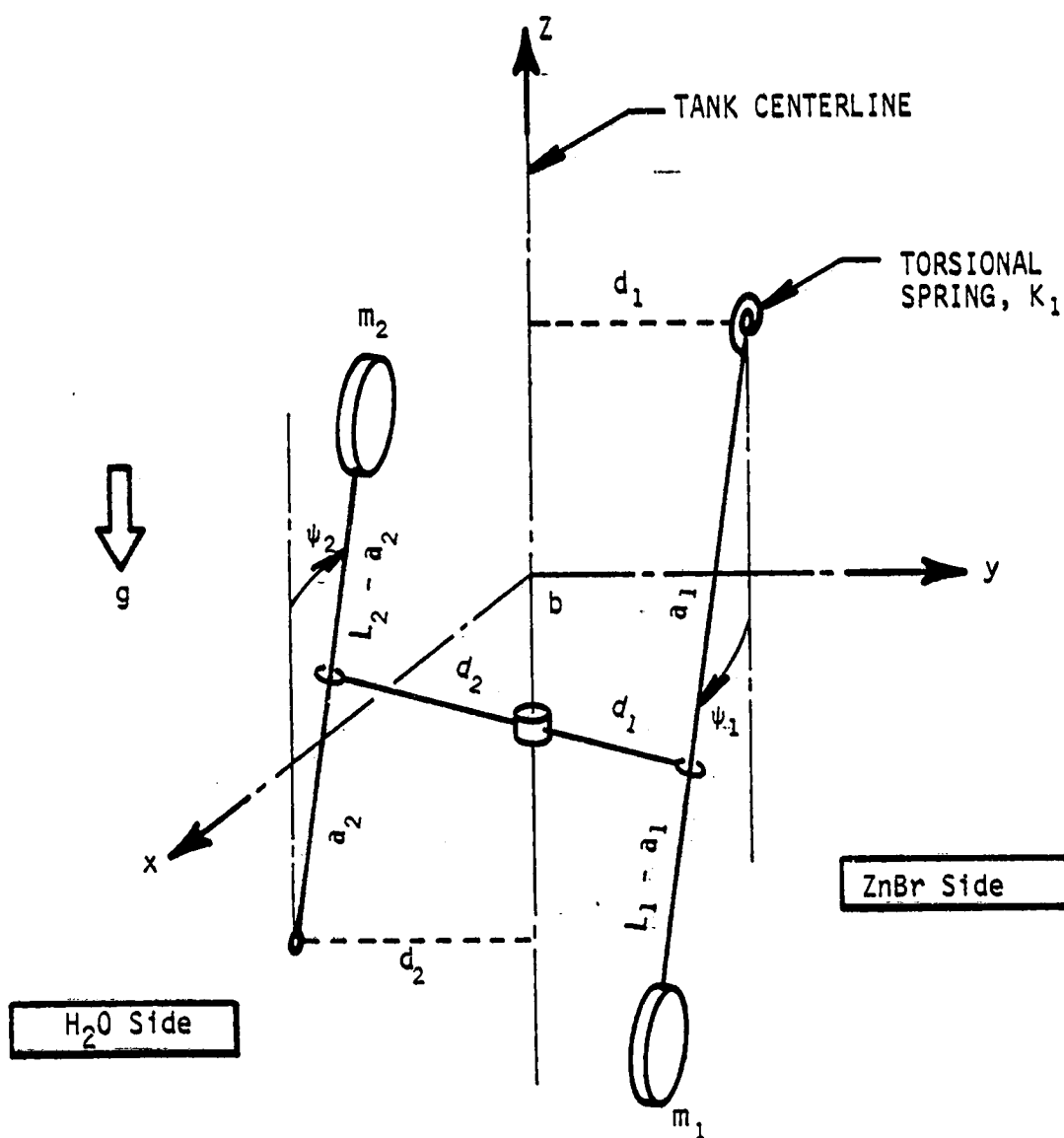


FIGURE C.1 SLOSH MODEL OF AFT TANK WITH LIQUID IN ULLAGE

Here \ddot{x}_1 is the oscillational acceleration (translation) of the zinc bromide pendulum pivot resulting from the tank motion, and \ddot{x}_2 is the corresponding acceleration of the water pendulum pivot. Combining these two equations gives

$$\begin{aligned} m_1 L_1^2 \ddot{\psi}_1 [1 + (a_1/a_2)(d_1/d_2)(L_2/L_1)(\rho_2/\rho_1)] \\ + \{K_1 + m_1 L_1 g [1 - (a_1/a_2)(d_1/d_2)(\rho_2/\rho_1)]\} \psi \\ = -m_1 L_1 [\ddot{x}_1 - (a_1/a_2)(d_1/d_2)(L_2/L_1)(\rho_2/\rho_1) \ddot{x}_2] \end{aligned} \quad (C.4)$$

When $\rho_2 = \rho_1$, the natural frequency of the system can depend only on the bladder stiffness (K_1) since unbalanced gravity forces are not possible then. This requires that

$$(a_1/a_2)(d_1/d_2) = 1 \quad (C.5)$$

In fact, equation (C.5) must always be true if volume of liquid on either side of the bladder remains constant because, since the bladder shape does not change, the other geometric parameters will also remain unchanged. Therefore, equation (C.4) can be simplified to give:

$$\begin{aligned} m_1 L_1^2 \ddot{\psi}_1 [1 + (L_2/L_1)(\rho_2/\rho_1)] + [K_1 + m_1 L_1 g (1 - \rho_2/\rho_1)] \psi \\ = -m_1 L_1 [\ddot{x}_1 - (L_2/L_1)(\rho_2/\rho_1) \ddot{x}_2] \end{aligned} \quad (C.6)$$

This equation can be used to rationalize the experimental observations.

When the zinc bromide was diluted to a specific gravity of 2.0, it was found (Figure B.18) that an x-axis translational excitation would not excite a slosh resonance. Since $\ddot{x}_1 = \ddot{x}_2$ for this form of excitation, it can be seen that the forcing will drop out of equation (C.6) if $(L_2/L_1) \times (\rho_2/\rho_1) = 1$, or $L_2 = 2L_1$. That is, the system "locks up" when $L_2 = 2L_1$. For rotation about the z-axis (Figure B.20) or about the y-axis (Figure B.22), \ddot{x}_1 and \ddot{x}_2 have opposite signs, however, and equation (C.6) predicts, therefore, that a resonance will be excited. The predicted natural frequency is

$$\begin{aligned} \omega_1 &= \{ [K_1 + 0.5 m_1 L_1 g] / [2 m_1 L_1^2] \}^{1/2} \\ &= 0.85 \text{ hz} \end{aligned} \quad (C.7)$$

since $m_1 = 288 \text{ lb}$. This is in excellent agreement with the observed 0.8 hz. The validity of obtaining a reduced gravity simulation by reducing the

density of the simulated propellant to near that of the water contained in the ullage is therefore confirmed. The results also provide an independent demonstration of the validity of the proposed correlation for the slosh spring constant.

In the tests for which the zinc bromide was further diluted to an s.g. of 1.25 (i.e., a simulated gravity 0.25 g's), a slosh resonance near 0.8 hz was observed for y-axis rotational excitation (Figure B.24). The slosh model predicts a natural frequency of

$$\begin{aligned}\omega_1 &= \left\{ [K_1 + 0.2 m_1 L_1 g] / [2.6 m_1 L_1^2] \right\}^{1/2} \\ &= 0.61 \text{ hz}\end{aligned}\tag{C.8}$$

since $m_1 = 180 \text{ lb}$. The predicted frequency is somewhat lower than the measured one. The prediction can be improved somewhat by letting L_2/L_1 be less than 2.0, although doing so will allow some degree of x-axis excitation in the predictions of the tests previously discussed. It might also be noted that a very large amplitude tank motion was required to produce measurable moments; the resulting nonlinearity might also have affected the resonant frequency.